Emergence is a formal theory about → James.

SCALE (spatial scale, temporal scale) → later in this talk.

and (more generally) → lecture 4.

HIERARCHY (under which falls certain kinds of modularity)

CSSS 2011 S. DeDeo
Monday June 26, 9 AM
http://samdeo.edu/csss
for additional resources.

- individuals in a group.
(individual to group cognition)
[& of course, group & supergroup)].

to use the example
[cooperative game theory]

- economic systems —
[individual decision within a coalition]
⇓
the barging between coalitions.
& so on.

- layers in a computational system (protocols for internet communication; multicore system — the "emergent computation" of ants)

* Introduction

* Symmetry
Permutations, Shifts, and Finite Group Theory
Invariances of Equations of Motion
Continuous Symmetries
Semigroups & “approximate” Symmetries

* Symmetry Breaking
Essential vs. Spontaneous
Navier-Stokes & Turbulent Symmetry Breaking
Symmetry Restored?

* Phase Transitions
Ising and XY Models
Annealing vs. Domain Wall Formation
Effective Theories for Defects

* Emergence Defined

* Signatures of Emergence in Animal Society

A General Definition of Emergence

A system has emergent properties when an effective theory of the system at some scale, or level of organization, is qualitatively different from the lower-level theory

effective theory
as derived from more fundamental theory
(distinct from just a model – we follow Weak emergence)

for us, here, we will use group theory and the notion of symmetry

question of scale (Jane's) – of level, more generally – lecture 4

Topics

Symmetry

Permutations, Shifts, and Finite Group Theory
Invariances of Equations of Motion
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Semigroups & "approximate" Symmetries

Symmetry Breaking

Essential *vs.* Spontaneous
Navier-Stokes & Turbulent Symmetry Breaking
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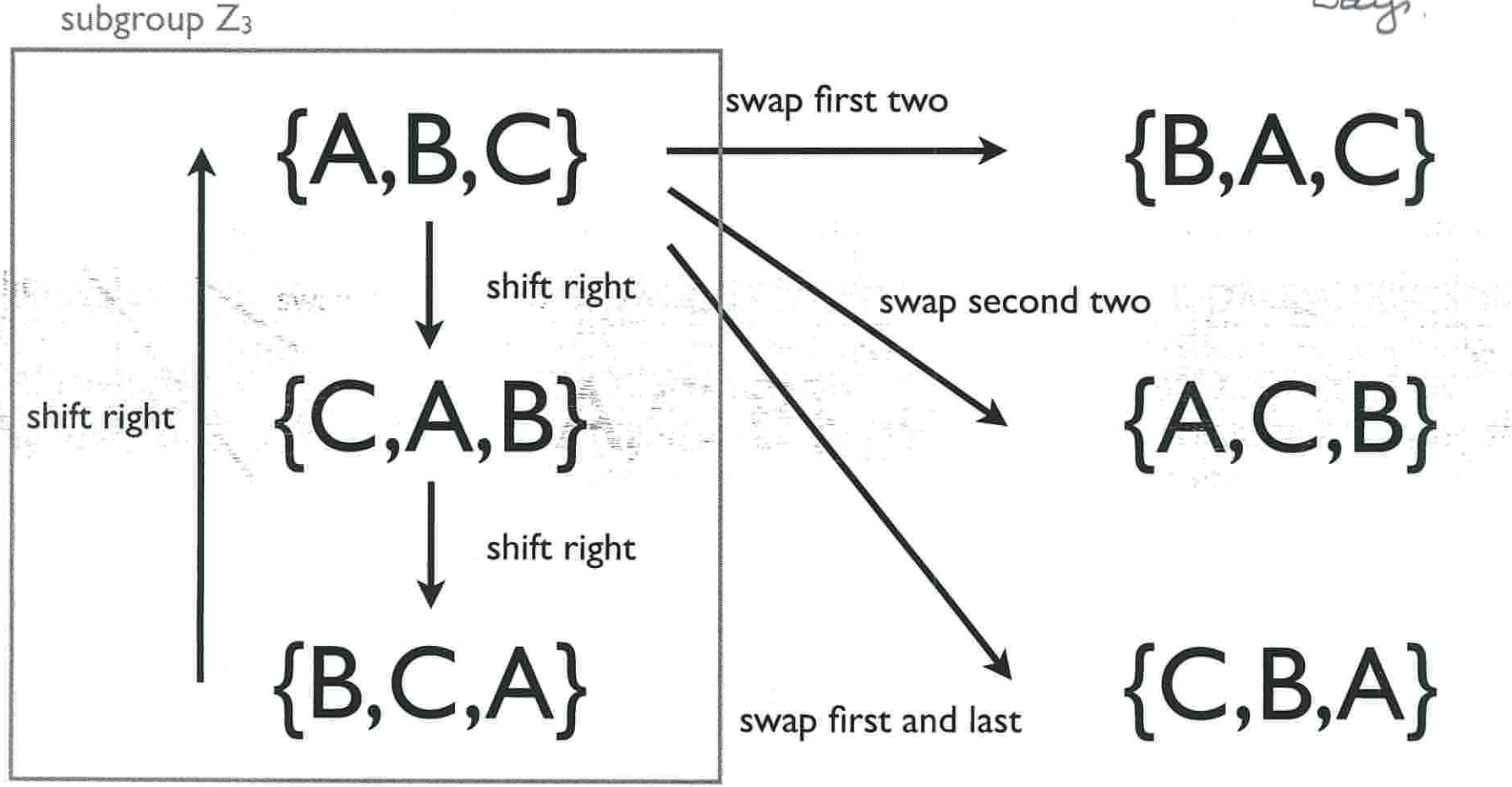
Symmetry Group

A set of object
↓
you can permute them in different ways



S_3 = {shift right, shift right twice, swap first two, shift right & swap first two, ...}

Symmetry Group

$$\sigma_i \in \{A, B, C\}$$

an equation of motion, or a function that you think describes how the system should evolve

$$H(\sigma_i, \sigma_j) = \begin{cases} 1 & \text{if } \sigma_i = \sigma_j \\ 0 & \text{otherwise} \end{cases}$$

may or may not preserve all the symmetry

$$H(\sigma_i, \sigma_j) \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

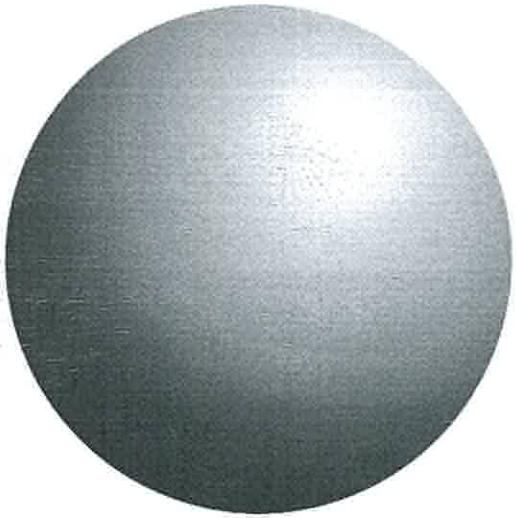
Essential Symmetry Breaking

$$H(\sigma_i, \sigma_j) \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

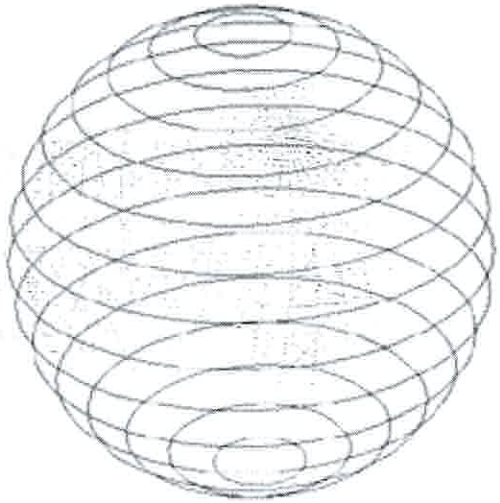
$$H(\sigma_i, \sigma_j) \sim \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$S_3 \rightarrow Z_2$

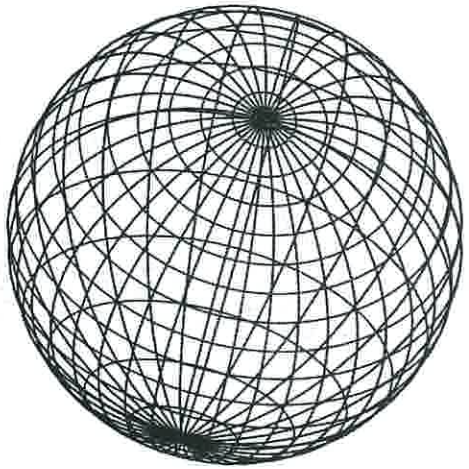
Continuous Symmetries



all rotations in 3 dimensions
a.k.a., O_3



only rotations in 2 dimensions
a.k.a., O_2



discrete rotations in 2 dimensions
a.k.a., Z_n
(where *n* is the number of longitude marks.)

$$O_3 \rightarrow O_2 \rightarrow Z_{2n}$$

Spontaneous Symmetry Breaking

Equations (and boundary conditions) governing the system have a symmetry

But the solution doesn't!