Emergence

Symmetry Breaking,
Topological Defects
and Effective Theories

Lecture 1
9 AM
27 June 2011

A General Definition of Emergence

A system has emergent properties when
an effective theory of the system at some scale,
or level of organization, is qualitatively different
from the lower-level theory

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an effective theory of the system at some scale,
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One Kind of Emergence

A system is emergent when the symmetries of the lower-level theory are violated under aggregation

Topics

Symmetry

Permutations, Shifts, and Finite Group Theory
Invariances of Equations of Motion
Continuous Symmetries
Semigroups & “approximate” Symmetries

Symmetry Breaking

Essential *vs.* Spontaneous
Navier-Stokes & Turbulent Symmetry Breaking
Symmetry Restored?

Phase Transitions

Ising and XY Models
Annealing *vs.* Domain Wall Formation
Effective Theories for Defects

Emergence Defined

Signatures of Emergence in Animal Society

Essential Symmetry Breaking

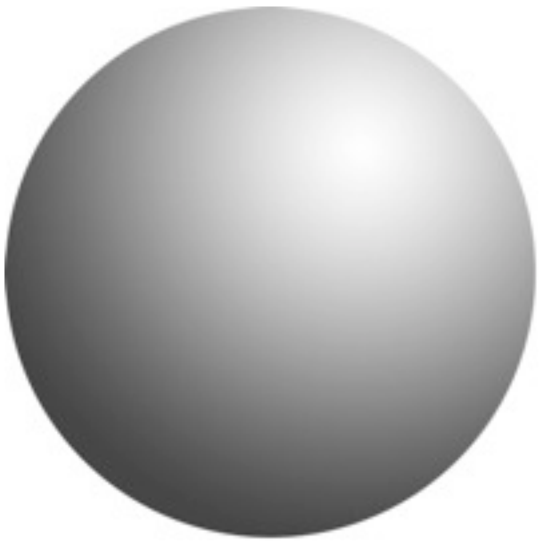
$$H(\sigma_i, \sigma_j) \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\downarrow$$

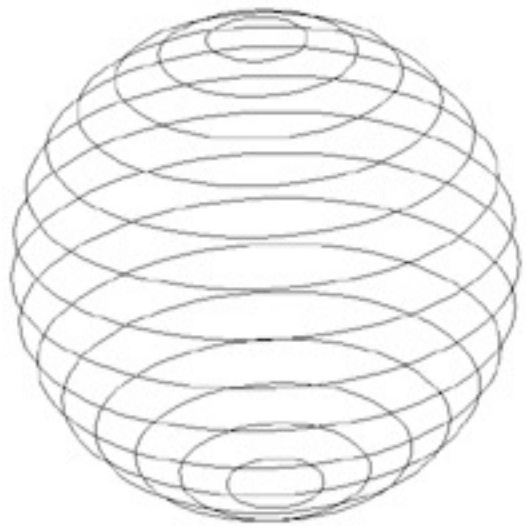
$$H(\sigma_i, \sigma_j) \sim \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$S_3 \rightarrow Z_2$

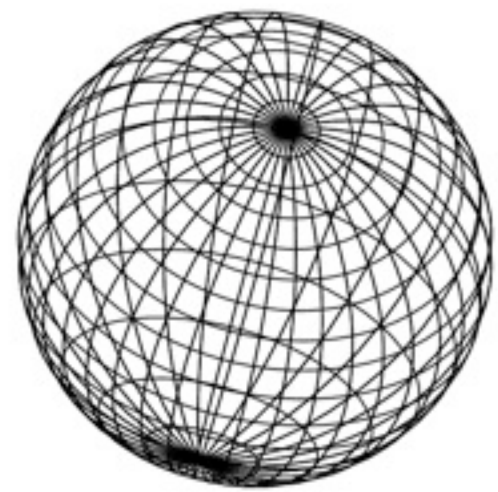
Continuous Symmetries



all rotations in 3 dimensions
a.k.a., O_3

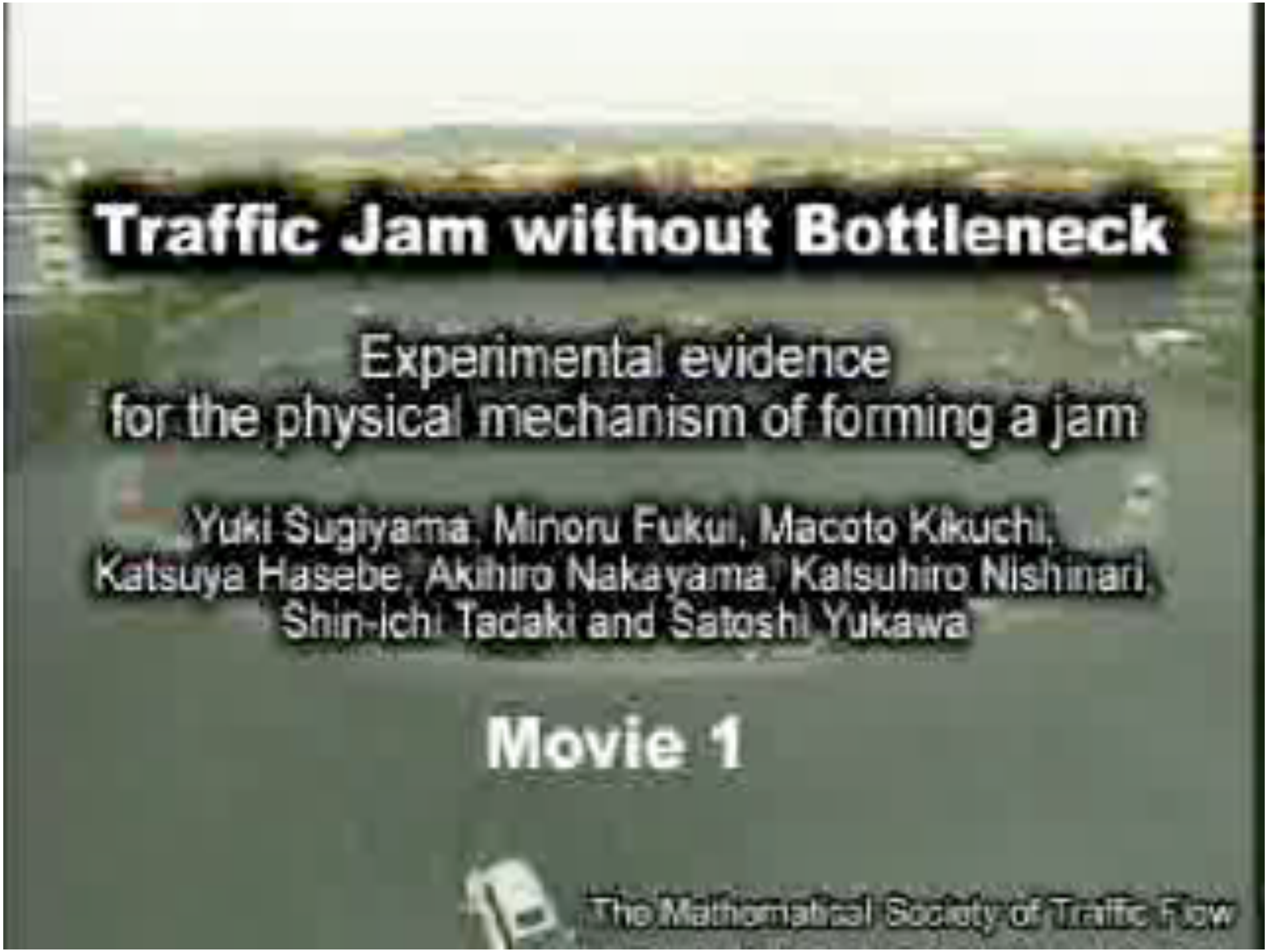


only rotations in 2 dimensions
a.k.a., O_2



discrete rotations in 2 dimensions
a.k.a., Z_n
(where *n* is the number of longitude marks.)

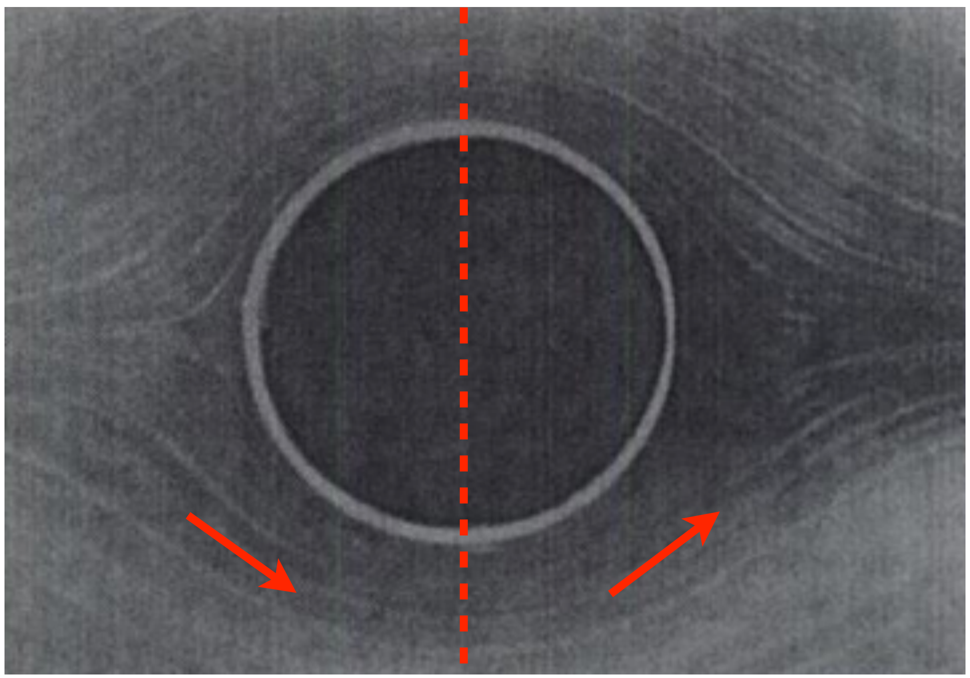
$O_3 \rightarrow O_2 \rightarrow Z_2$


Traffic Jam without Bottleneck
Experimental evidence
for the physical mechanism of forming a jam
Yuki Sugiyama, Minoru Fukui, Macoto Kikuchi,
Katsuya Hasebe, Akihiro Nakayama, Katsuhiro Nishinari,
Shin-ichi Tadaki and Satoshi Yukawa
Movie 1
The Mathematical Society of Traffic Flow

Navier-Stokes Equation

$$\frac{\partial v_i}{\partial t} \qquad = -\frac{\partial p}{\partial x_i} + \nu \sum_j \frac{\partial^2}{\partial x_j^2} v_i$$

Symmetry: flip *x* position, *y* velocity



Uriel Frisch, *Turbulence*

Navier-Stokes Equation

$$\frac{\partial v_i}{\partial t} + \sum_j v_j \frac{\partial}{\partial x_j} v_i = -\frac{\partial p}{\partial x_i} + \nu \sum_j \frac{\partial^2}{\partial x_j^2} v_i$$

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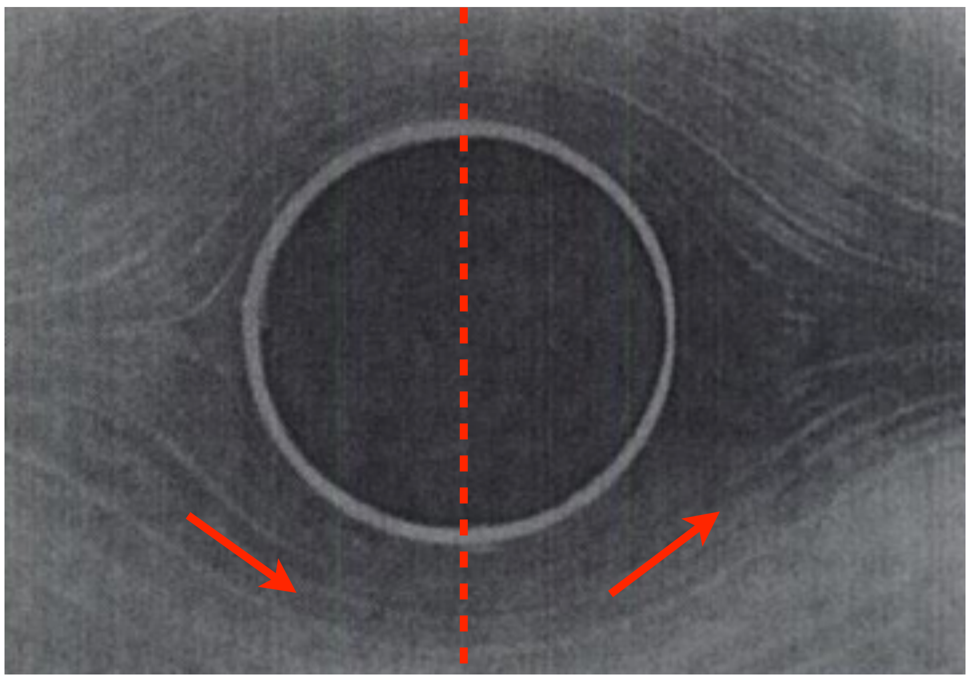

Uriel Frisch, *Turbulence*

Navier-Stokes Equation

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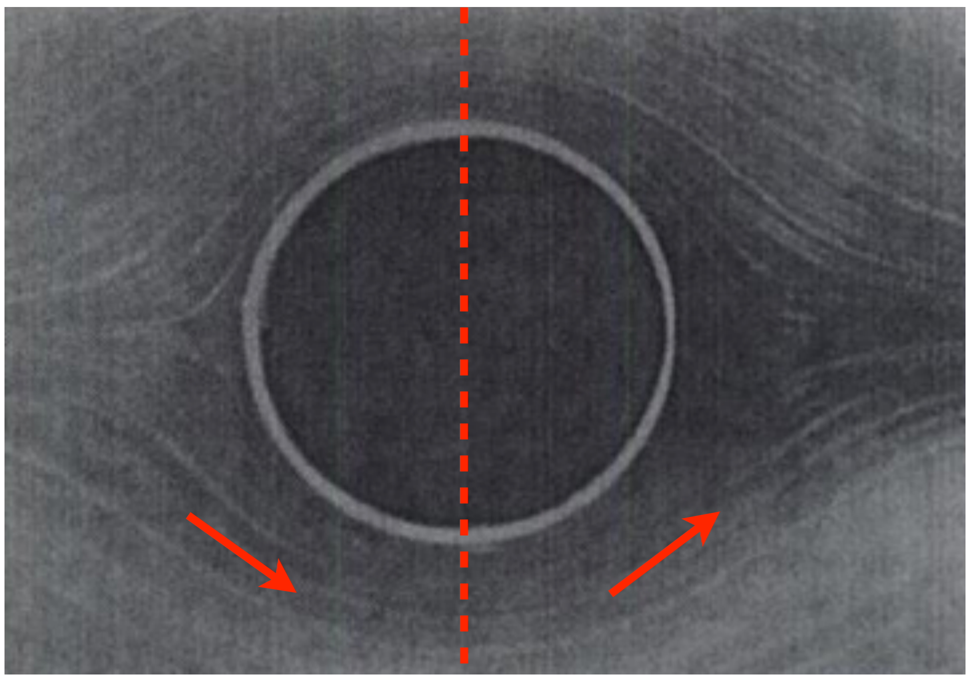
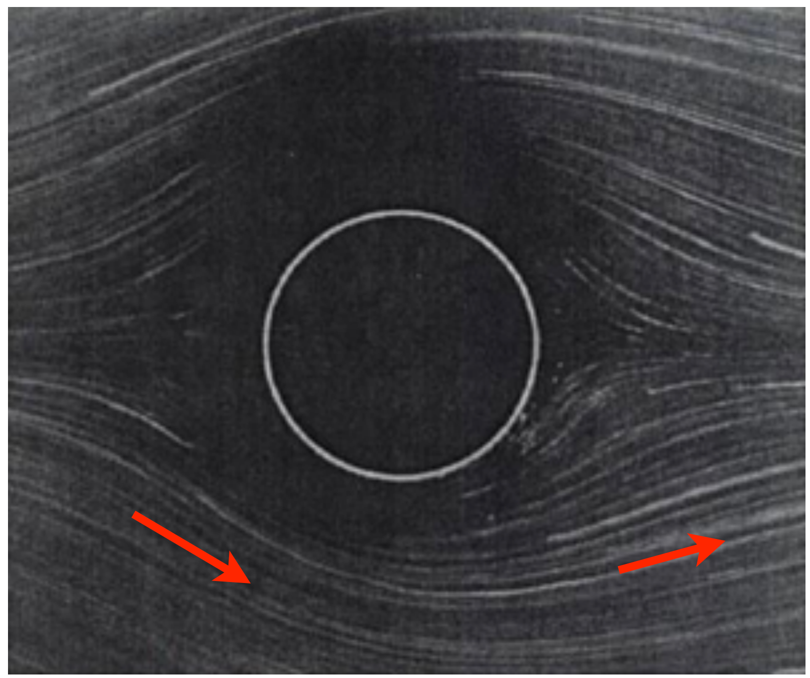


...upon turning on non-linear term



Uriel Frisch, *Turbulence*

Navier-Stokes Equation

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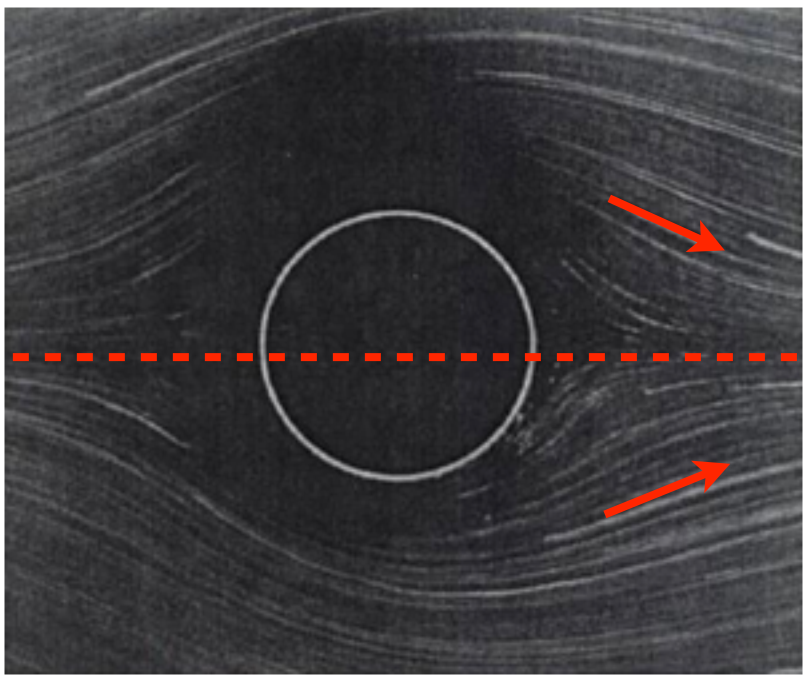
y symmetry still exists...



Navier-Stokes Equation

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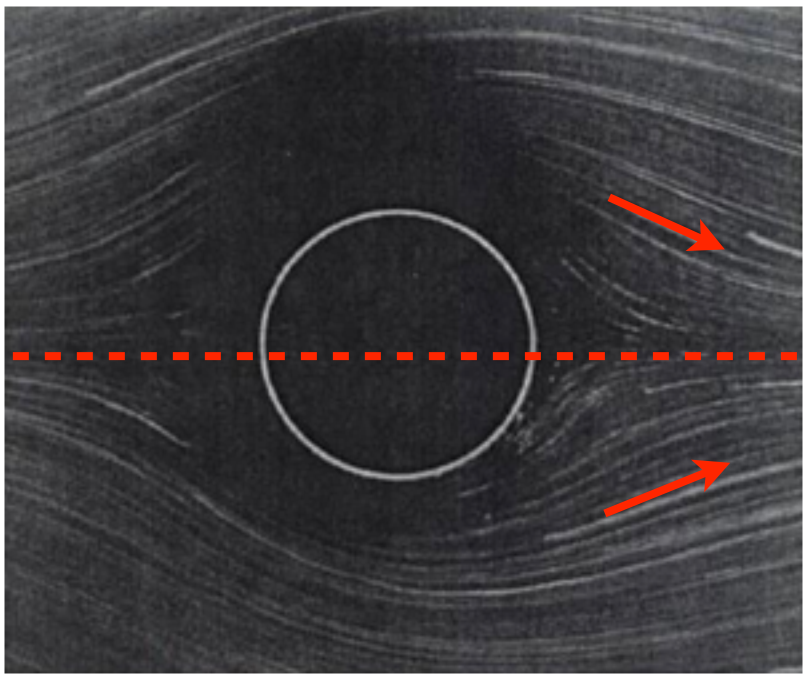
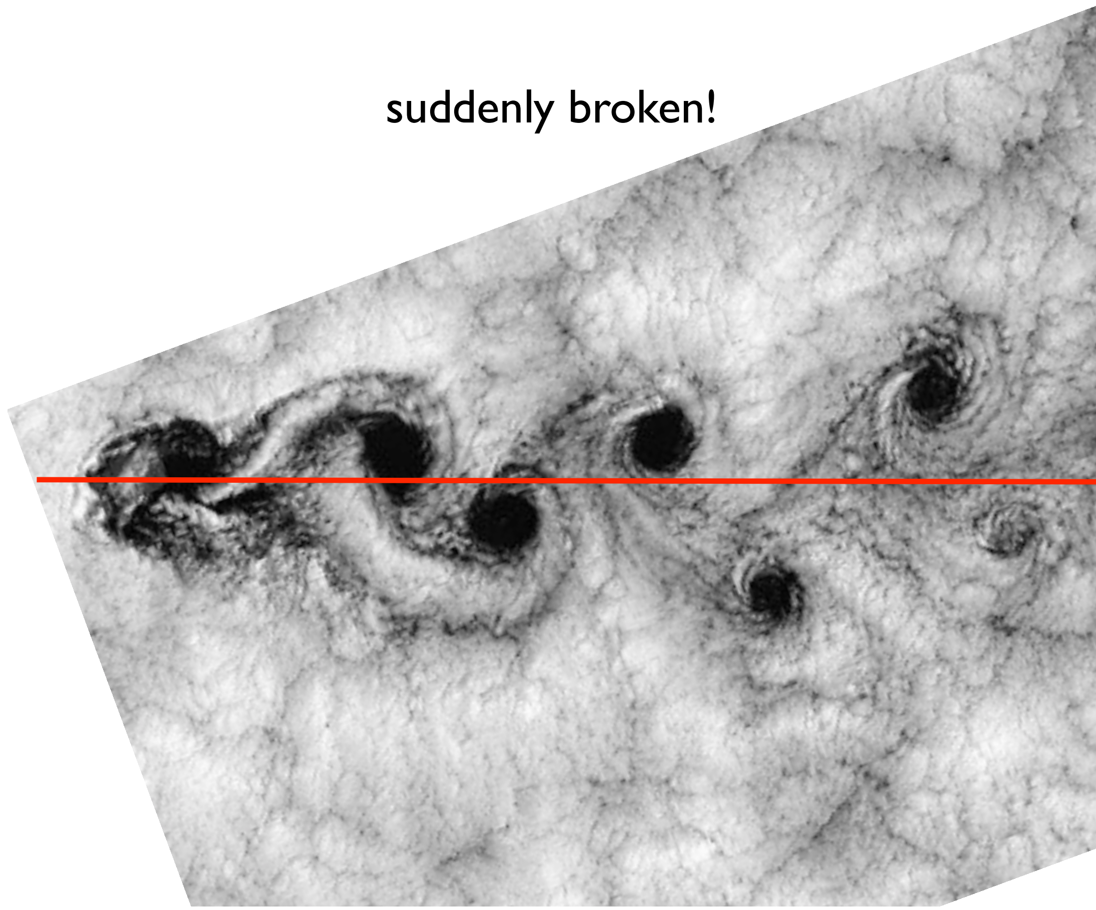


suddenly broken!



Control Parameter

Characteristic Velocity and Length Scales

Reynold's Number

$$\mathrm{Re} = \frac{L\ V}{\nu}$$

System Dissipation

Pablo Navarrete Michelini, University of Chile

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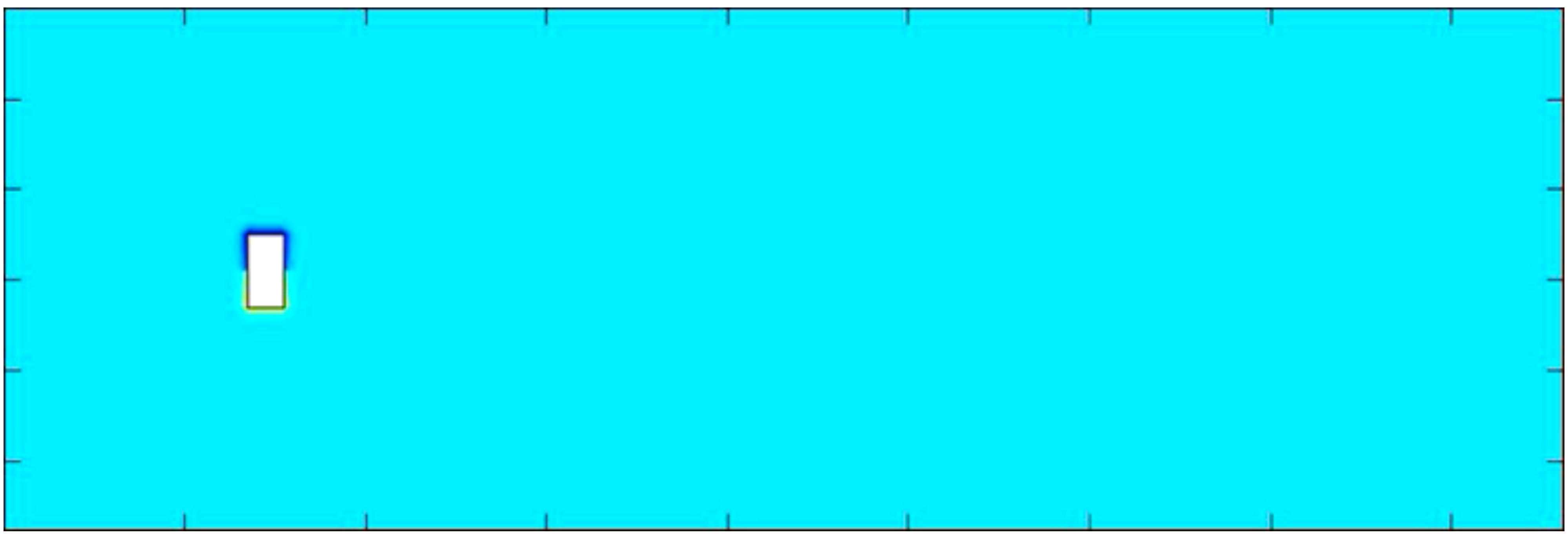


Pablo Navarrete Michelini, University of Chile

What is your symmetry?

discrete? Fully symmetric? Cyclic? A pair of cycles? A set of Nash Equilibria?
continuous? A real number? A vector?
[a fundamental mechanism question]

What is your interaction?

pair wise? (neighbours on a graph, cars in a line?) Mean field? (Well mixed population?)
Higher-order? (Cooperative effects, spatial structures...)
[a fundamental mechanism question]

What is your control parameter?

Noise? Dissipation? Penalty for failure?
Strength of interaction? Of pairwise:triplet interaction ratio?
[when the system shows emergence]

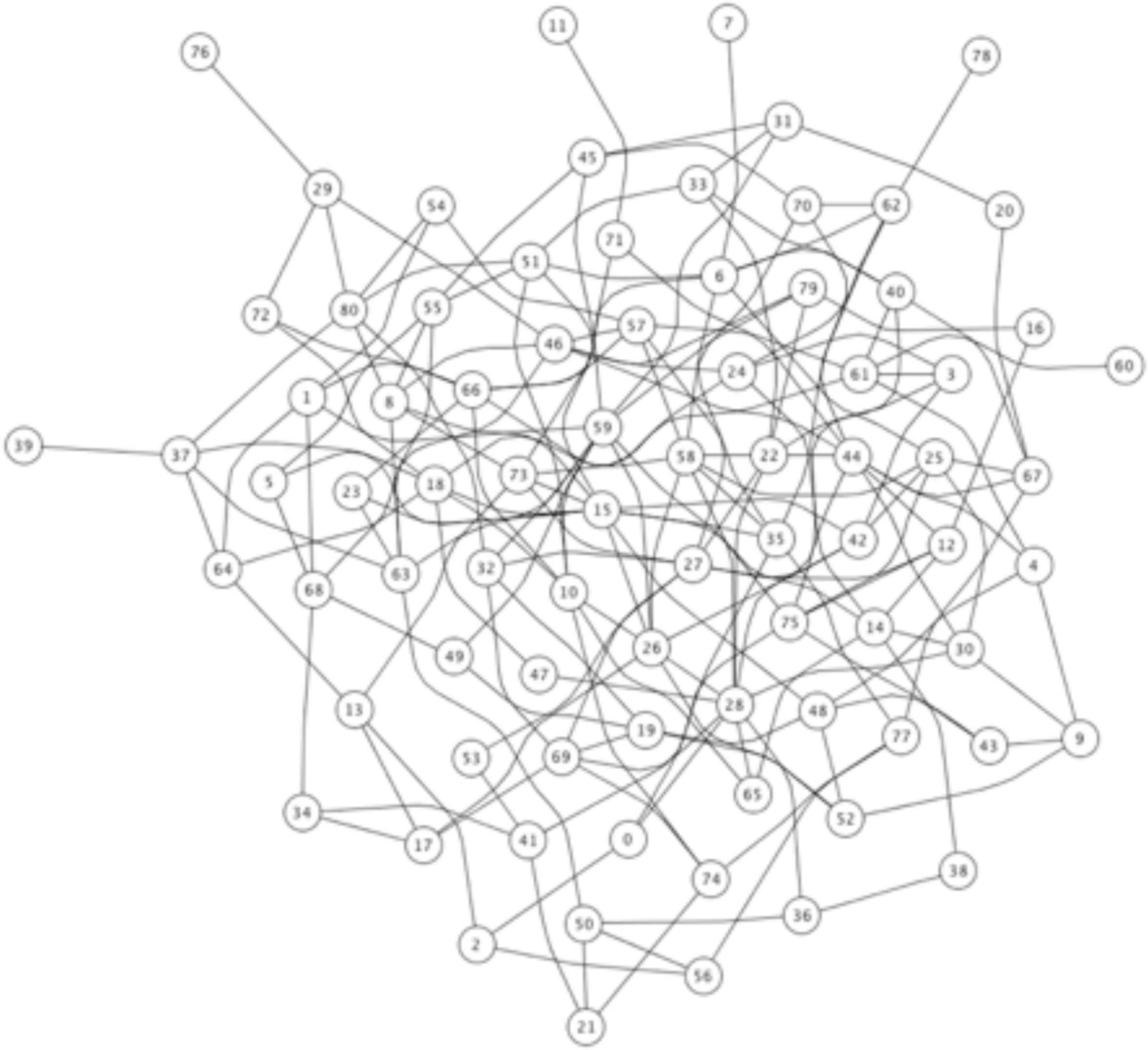
Broken Symmetries
& Topological Defects

The Cost Function of the Ising Model

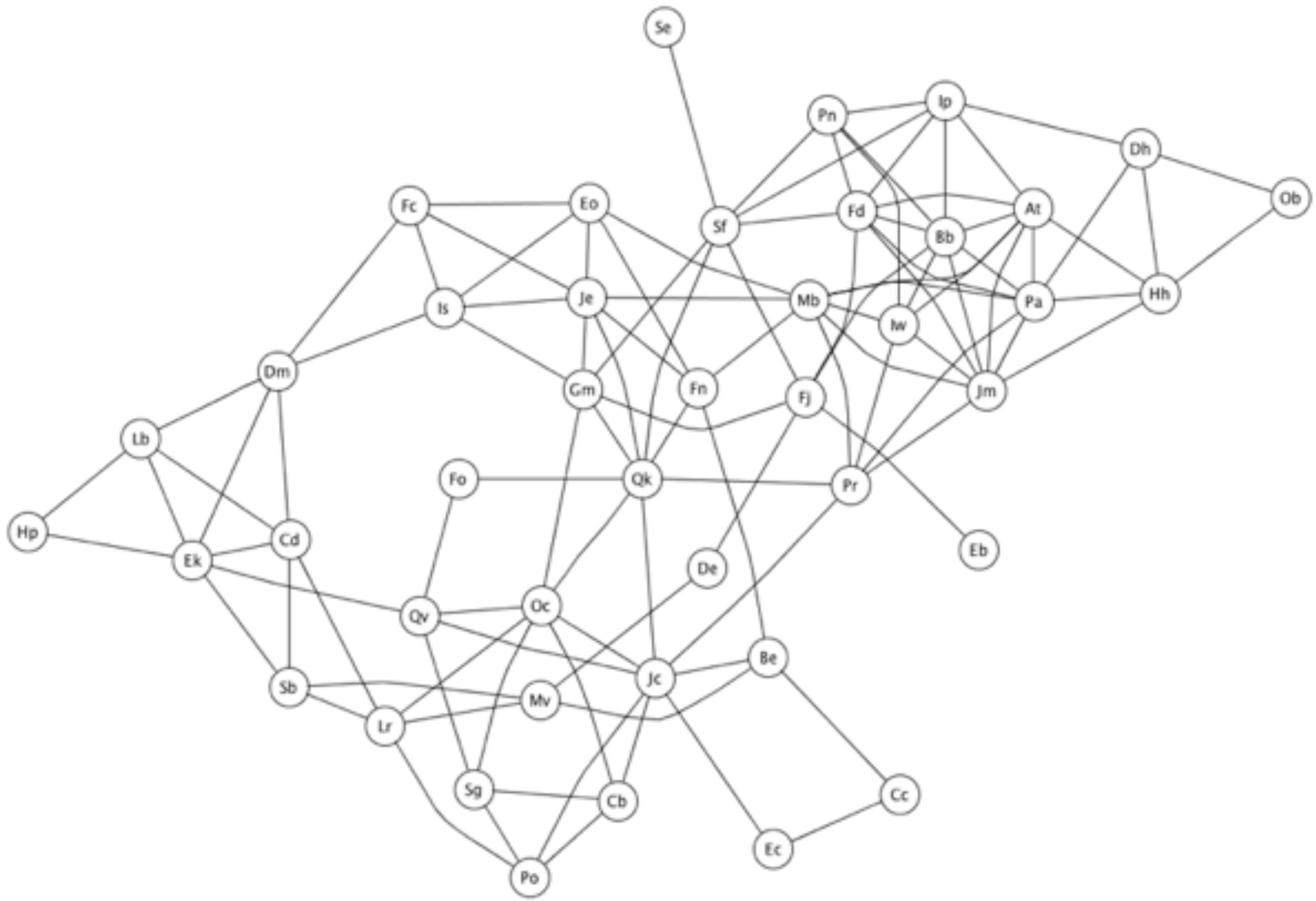
$$H[\{\sigma_i\}] = -\sum_{i,j} J_{ij}\sigma_i\sigma_j$$

Z_2 symmetry (flip all +1 to -1, and vice-versa)

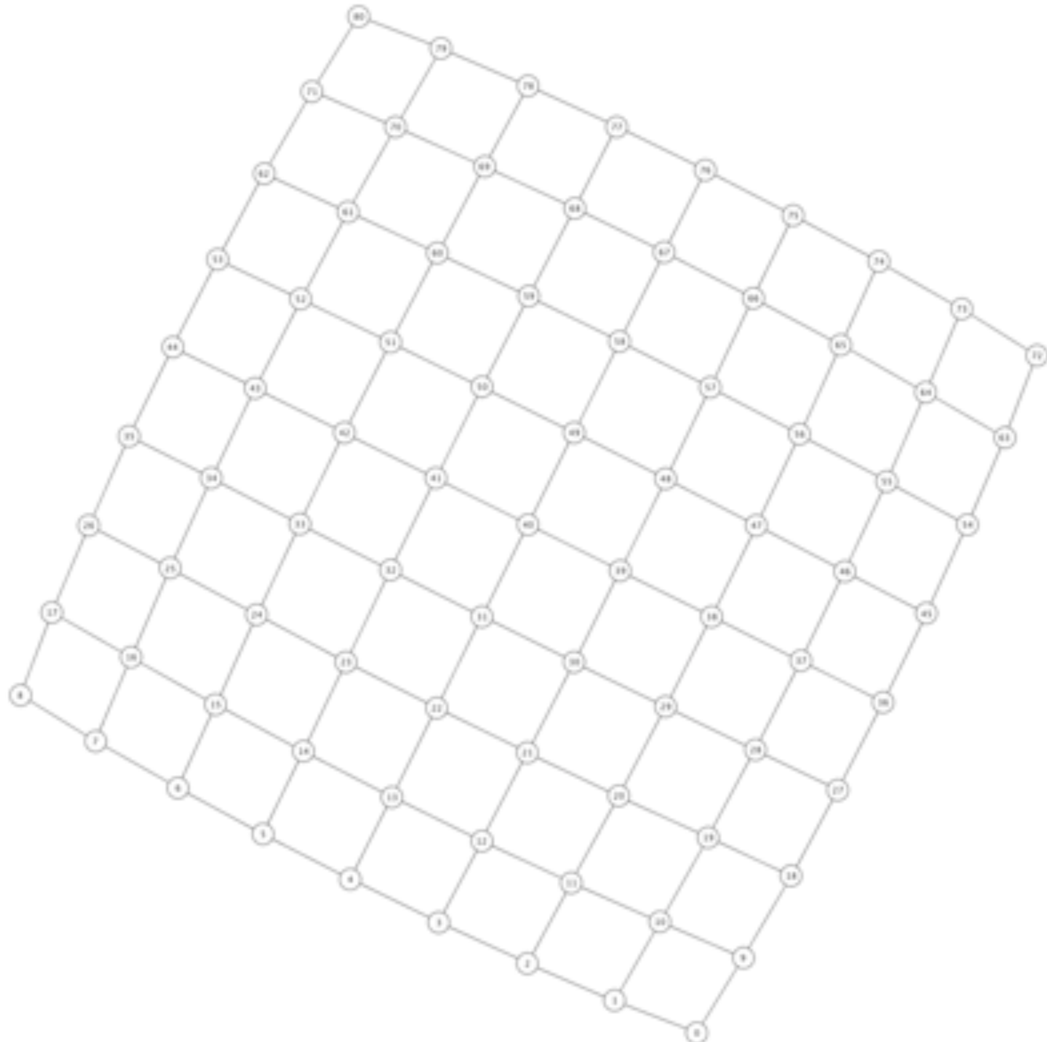
Erdős-Renyi Random Graph



Pigtail Macaques at Emory



Lattice



Ising Summary

What is your symmetry?

Each node can be "up" or "down", and the cost is invariant if you flip everyone at once (Z_2 symmetry)

What is your interaction?

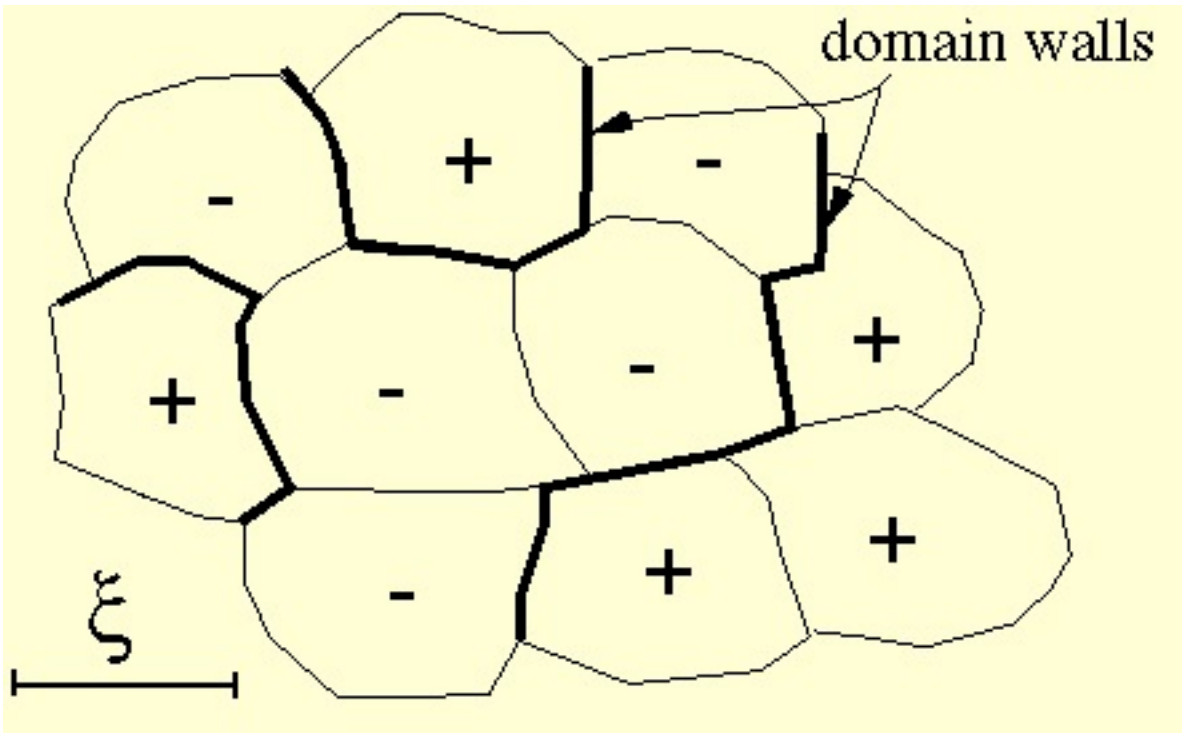
pairwise interactions between neighbours on a lattice, that promote sameness (nodes want to be in the same state)

What is your control parameter?

noise ("temperature") — higher noise means influence of neighbours is small

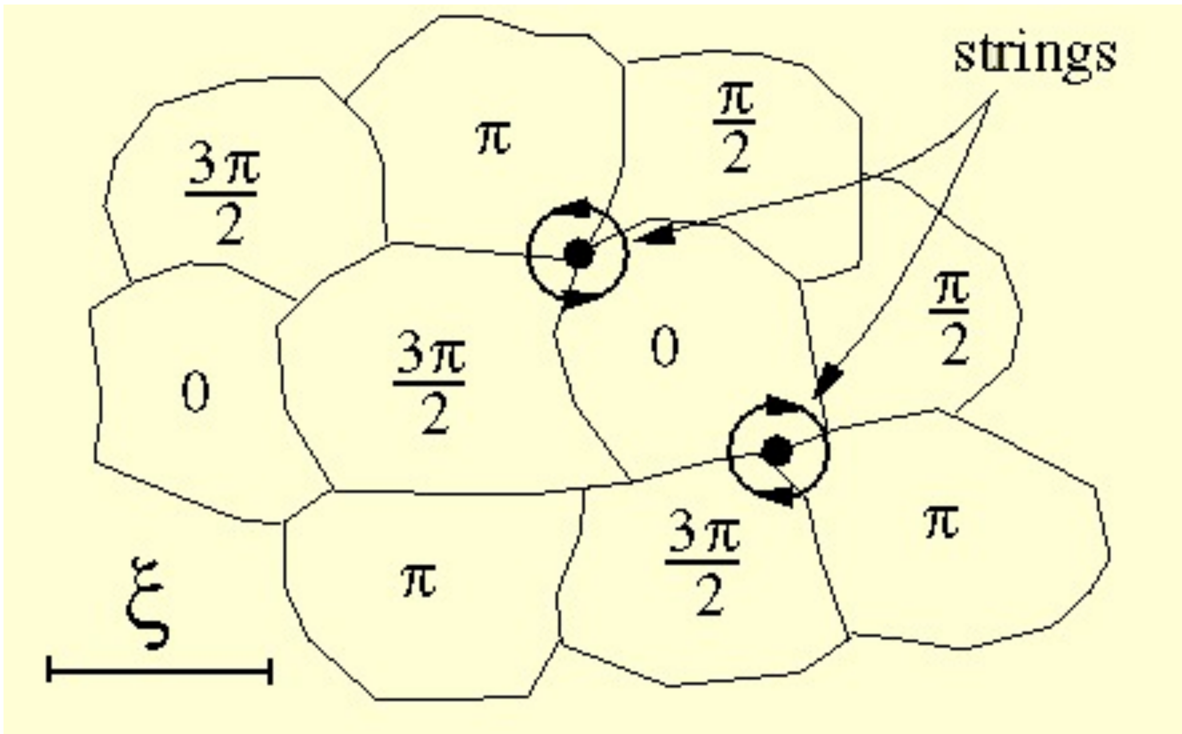
[Ising Simulation]

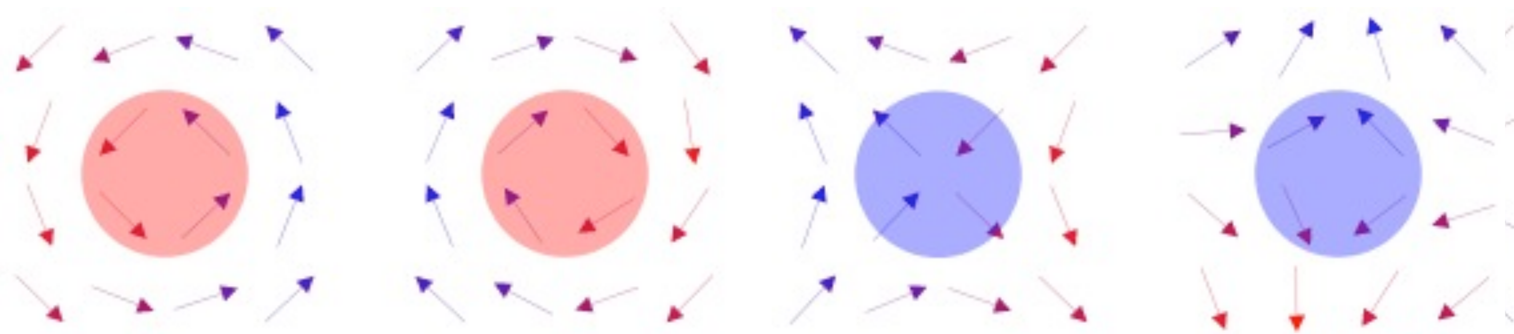
Topological Defects



Cambridge / DAMTP
(see E.P. Shellard et al.)

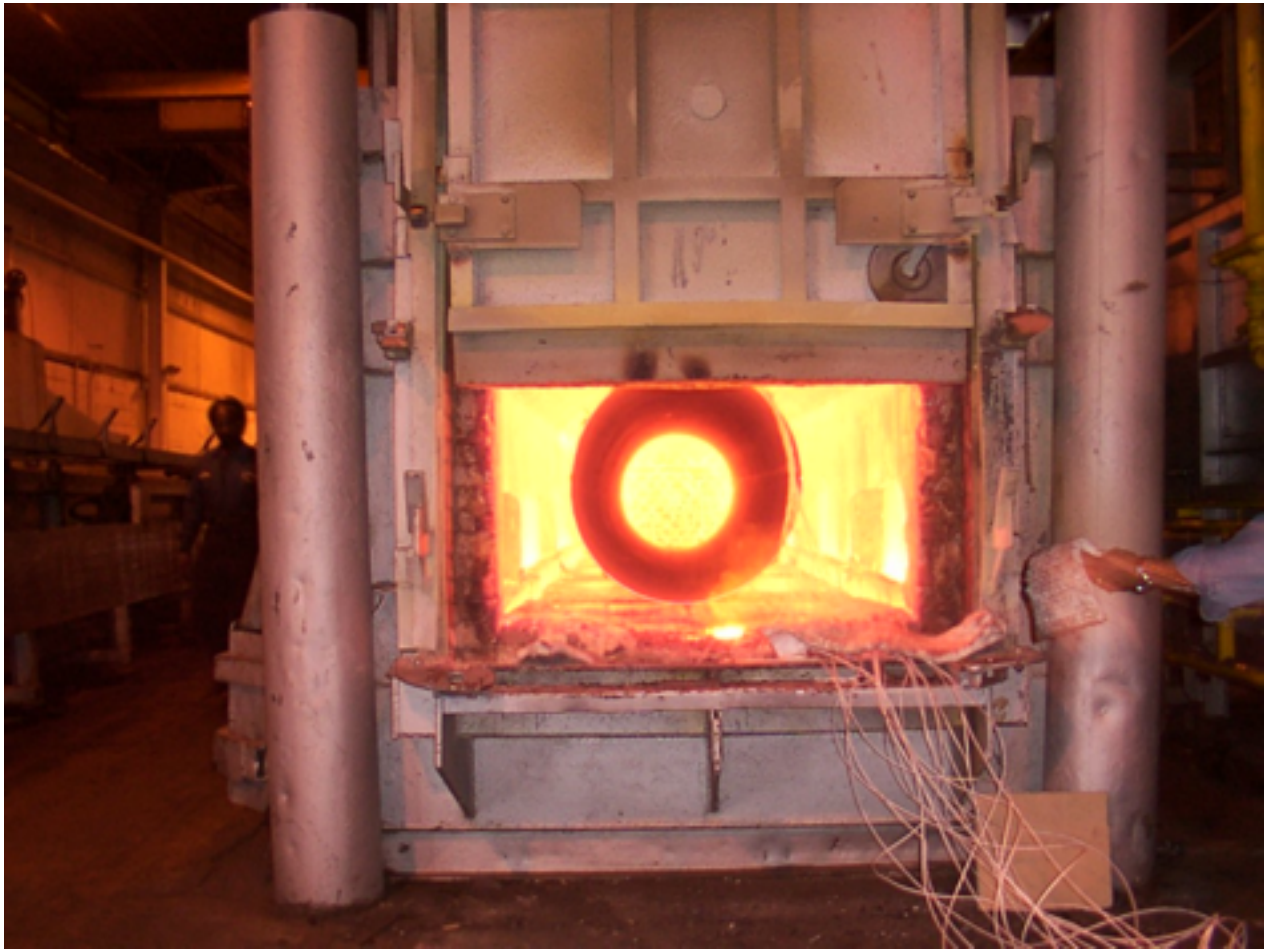
More Elaborate Symmetries





Vortex Images: Käser, Maier & Rautenkranz (2007)

Annealing



many defects:
(quenched)
strong, but brittle
— hard to cleave

few defects:
(annealed)
ductile, flexible


NYSE
NYSE
NYSE
NYSE
NYSE
CYPRESS
10
11


سقط مبارك

What is your symmetry?

discrete? Fully symmetric? Cyclic? A pair of cycles? A set of Nash Equilibria?
continuous? A real number? A vector?
[a fundamental mechanism question]

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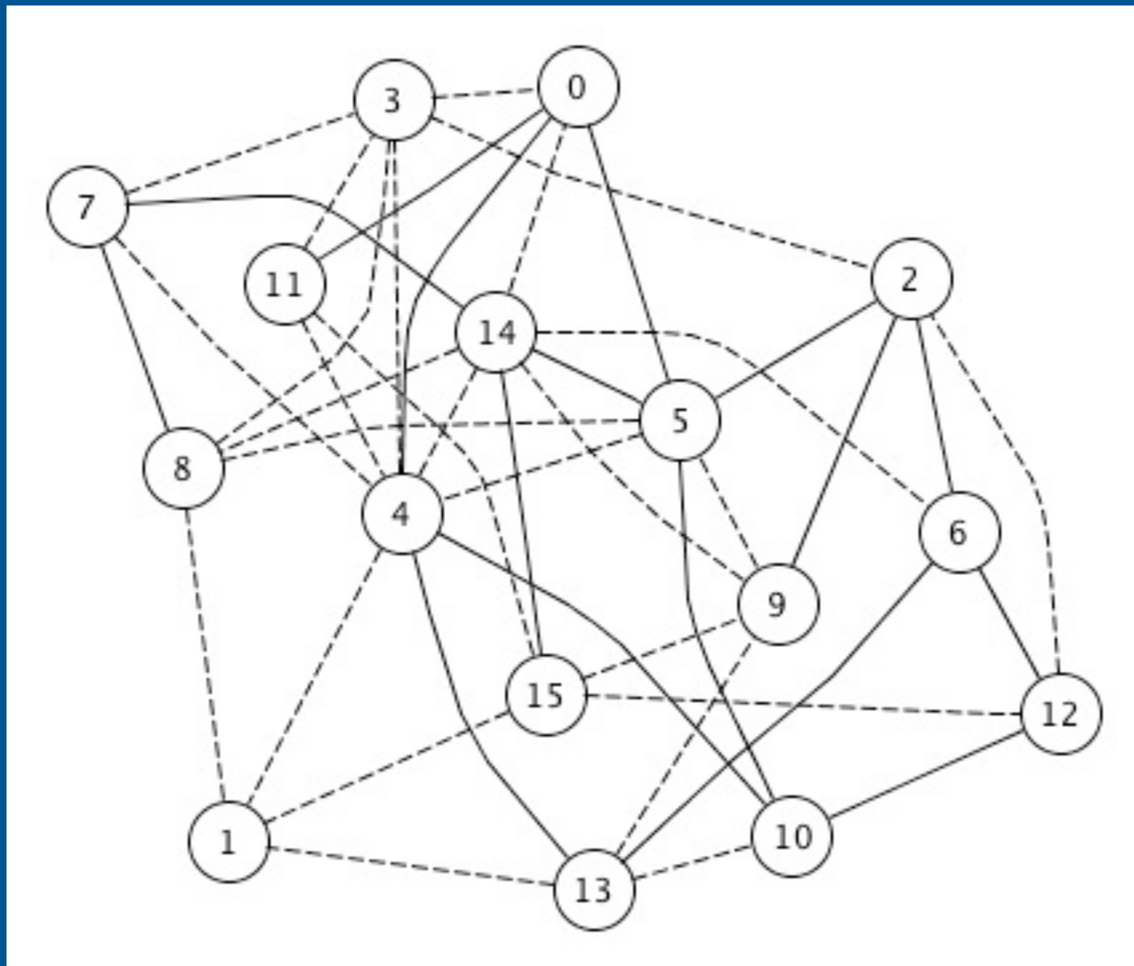
What is your control parameter?

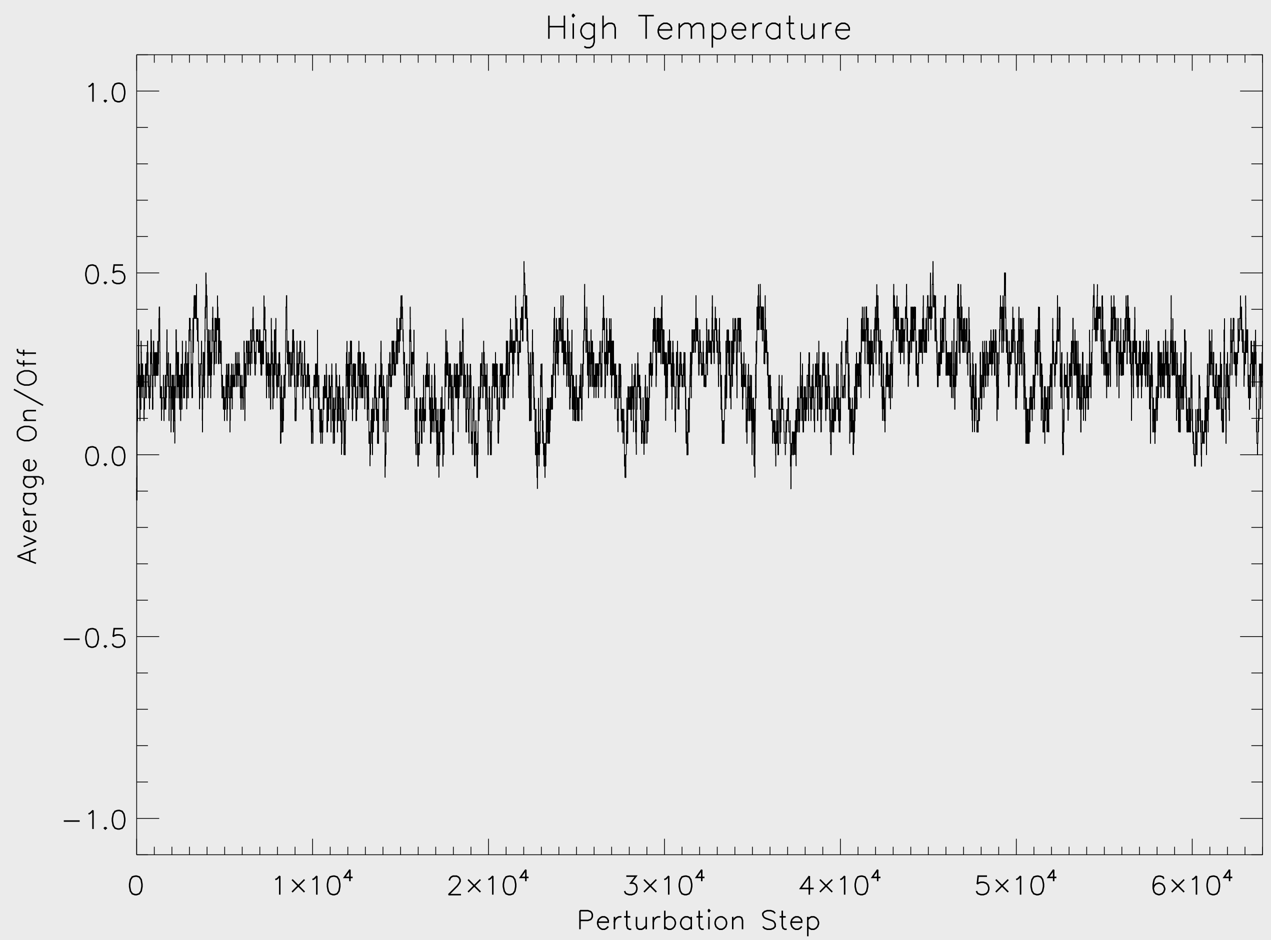
Noise? Dissipation? Penalty for failure?
Strength of interaction? Of pairwise:triplet interaction ratio?
[when the system shows emergence]

updated worksheet: check wiki, or http://santafe.edu/~simon/practical.pdf

the Spin Glass



Memory & the Rugged Landscape



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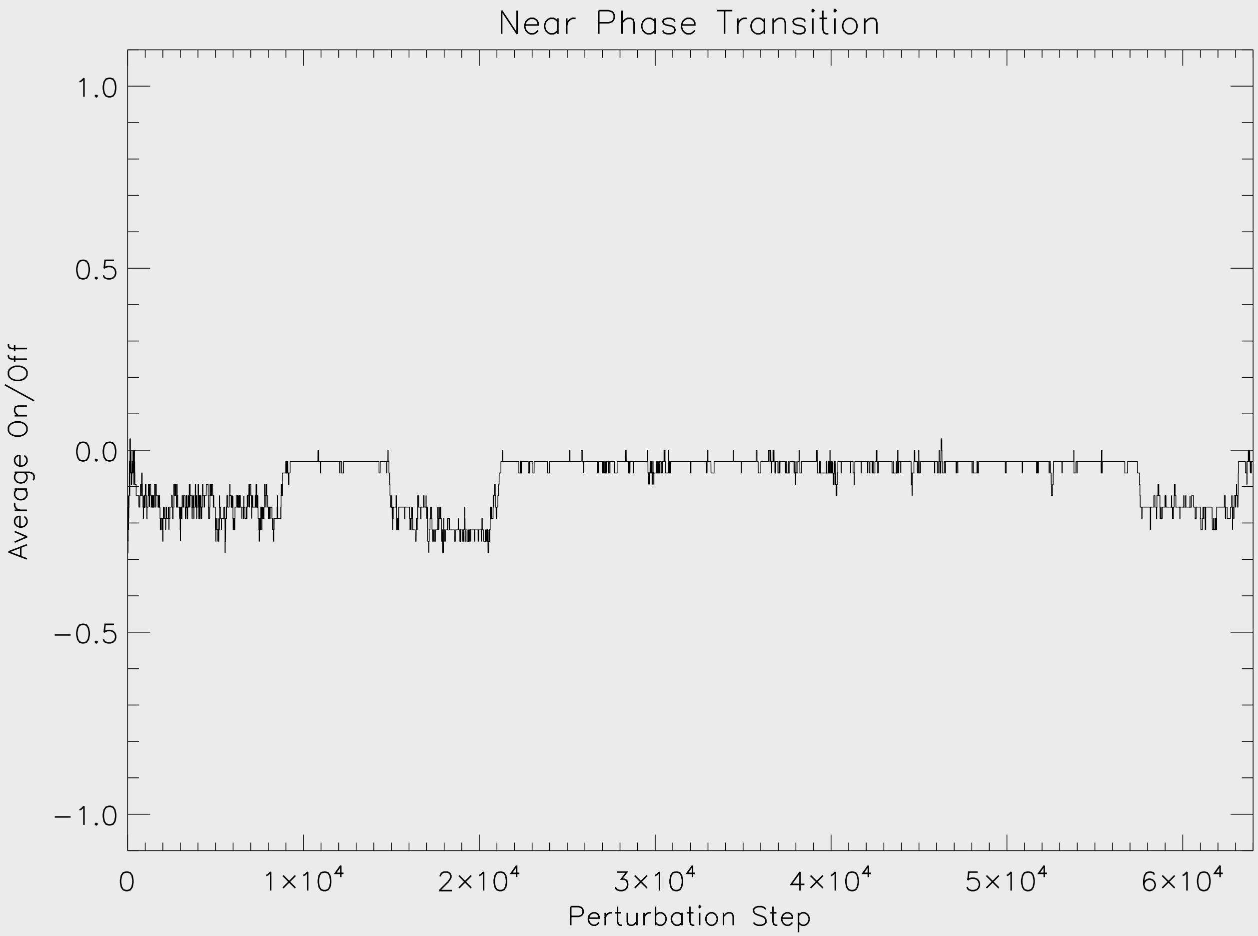

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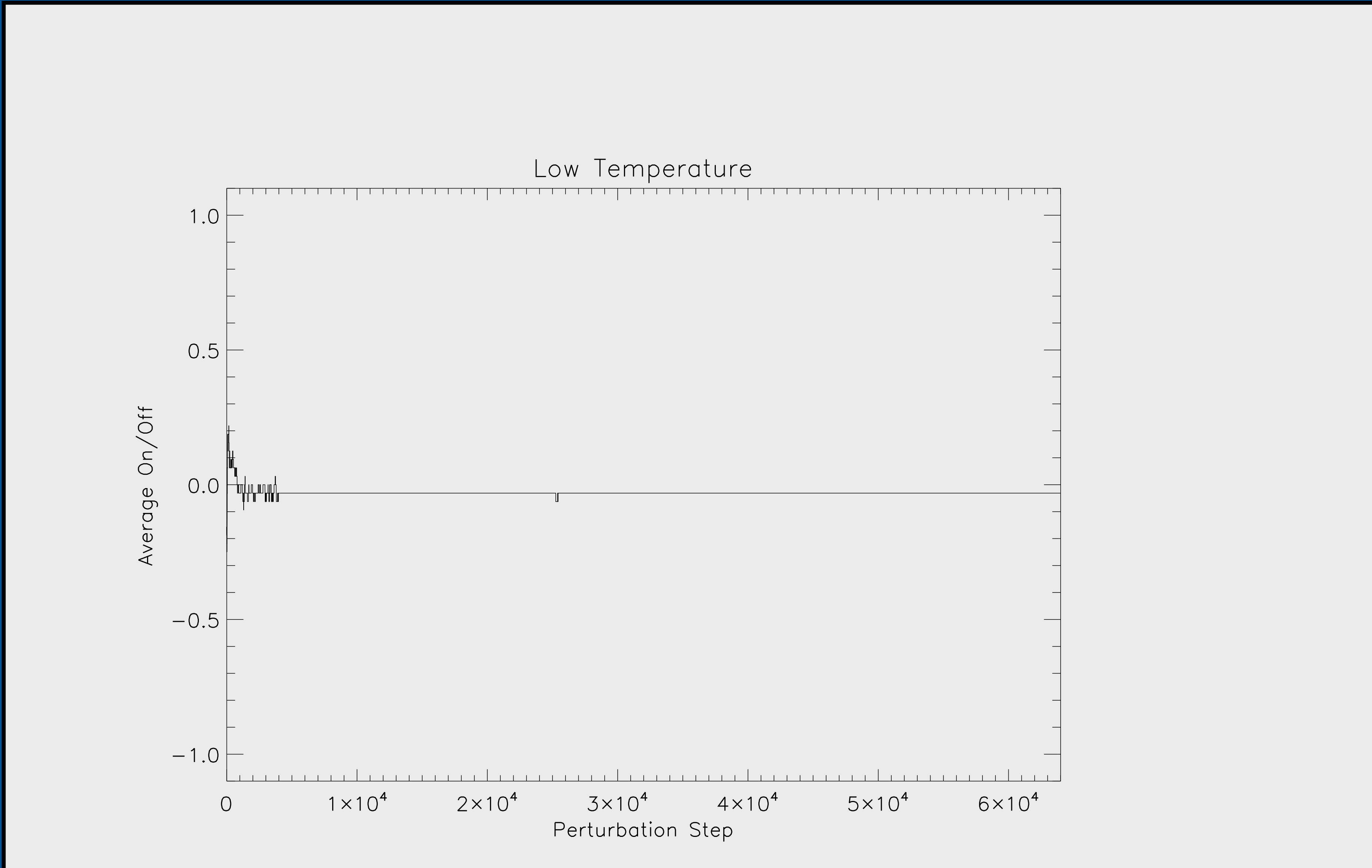

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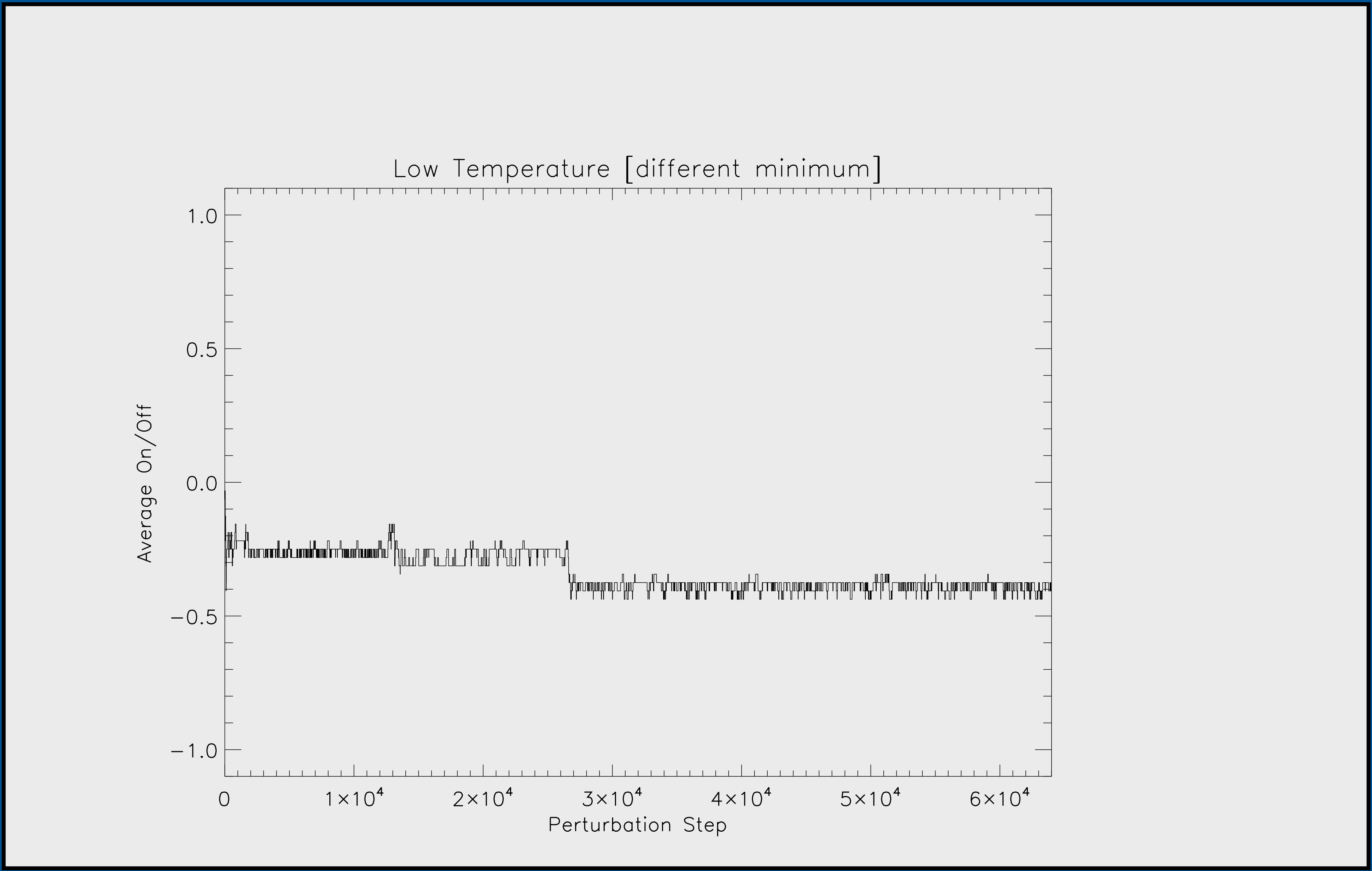

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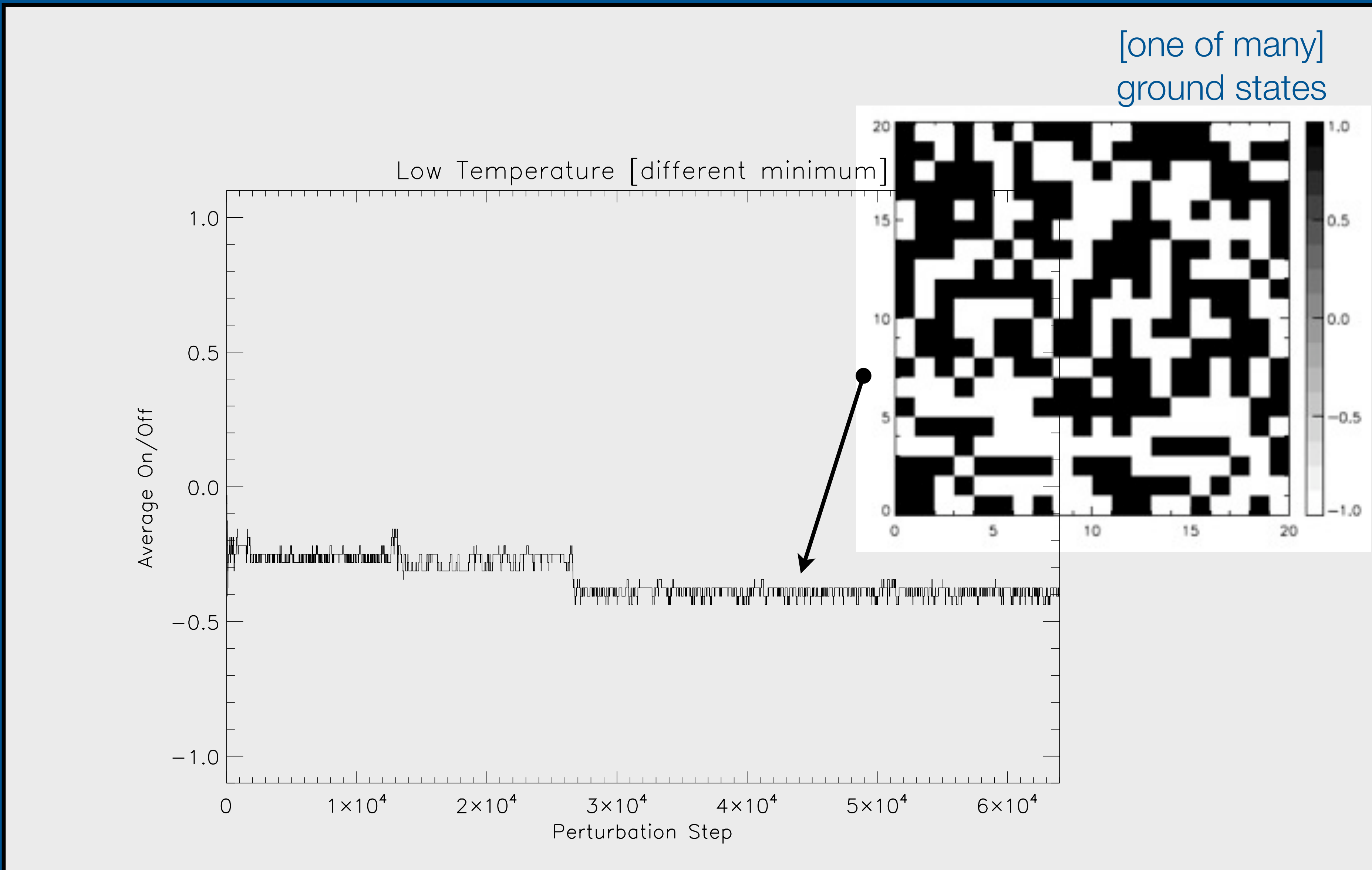
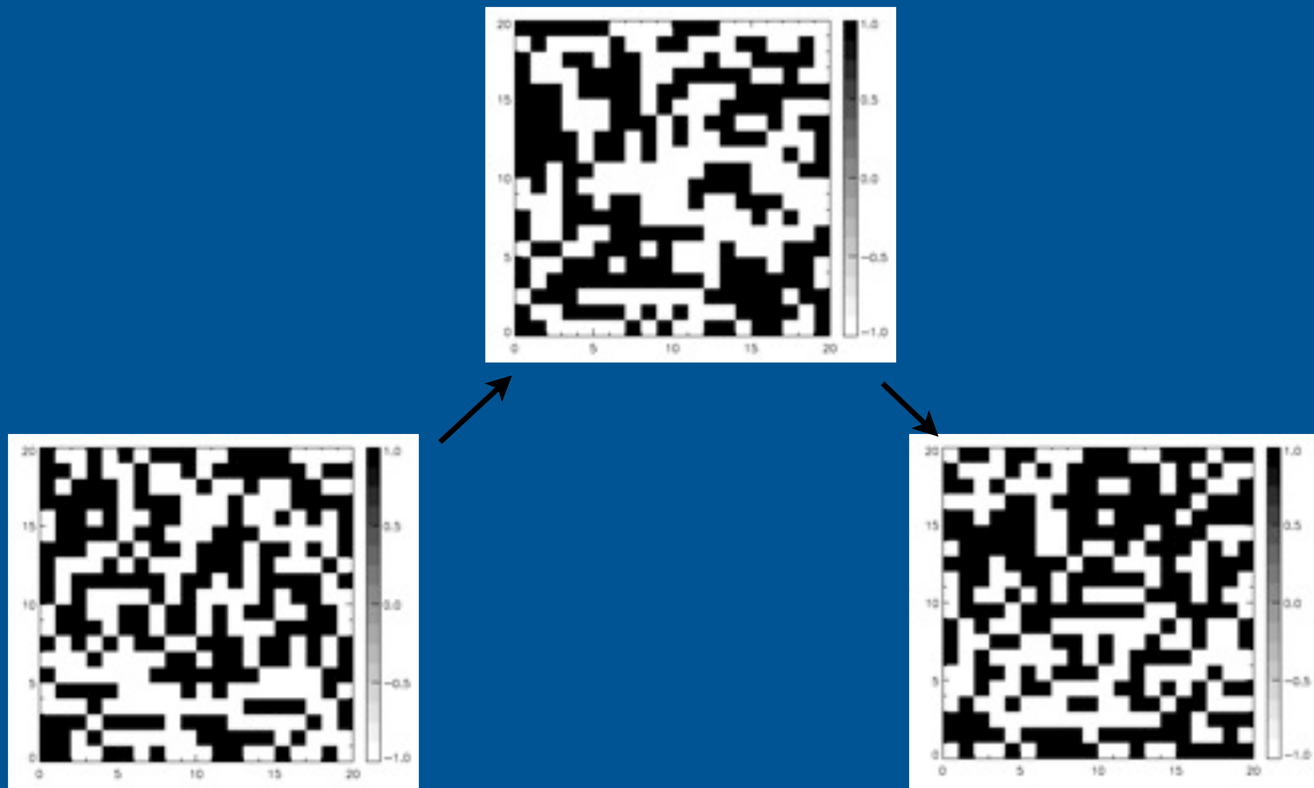


Sickness & Health

high-temperature



Salamanders!

Ising models for networks of real neurons

Gašper Tkačik,[a,c] Elad Schneidman,[a−c] Michael J. Berry II,[b] and William Bialek[a,c]
[a]Joseph Henry Laboratories of Physics, [b]Department of Molecular Biology,
and [c]Lewis–Sigler Institute for Integrative Genomics
Princeton University, Princeton, New Jersey 08544 USA
(Dated: February 4, 2008)

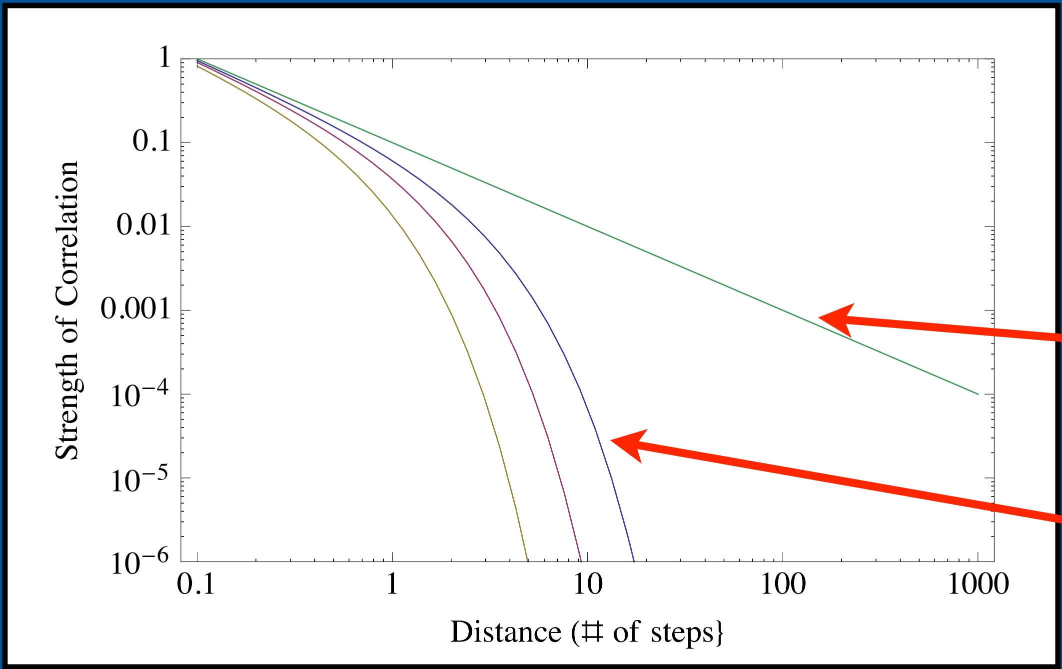
Ising models with pairwise interactions are the least structured, or maximum–entropy, probability distributions that exactly reproduce measured pairwise correlations between spins. Here we use this equivalence to construct Ising models that describe the correlated spiking activity of populations of 40 neurons in the retina, and show that pairwise interactions account for observed higher–order correlations. By first finding a representative ensemble for observed networks we can create synthetic networks of 120 neurons, and find that with increasing size the networks operate closer to a critical point and start exhibiting collective behaviors reminiscent of spin glasses.

PACS numbers: 87.18.Sn, 87.19.Dd, 89.70.+c

Physicists have long explored analogies between the statistical mechanics of Ising models and the functional dynamics of neural networks [1, 2]. Recently it has been suggested that this analogy can be turned into a precise mapping [3]: In small windows of time, a single neuron i either does ($\sigma_i = +1$) or does not ($\sigma_i = -1$) generate an action potential or "spike" [4]; if we measure the mean

ments of Refs [3, 7]. Under these conditio
cells within $\sim 200\,\mu$m of each other have
drawn from a homogeneous distribution; the
decline at larger distance [8]. This correlated
tains $N \sim 200$ neurons, of which we record fr
[9]; experiments typically run for ~ 1 hr [10].
The central problem is to find the magneti

Critical Opalescence



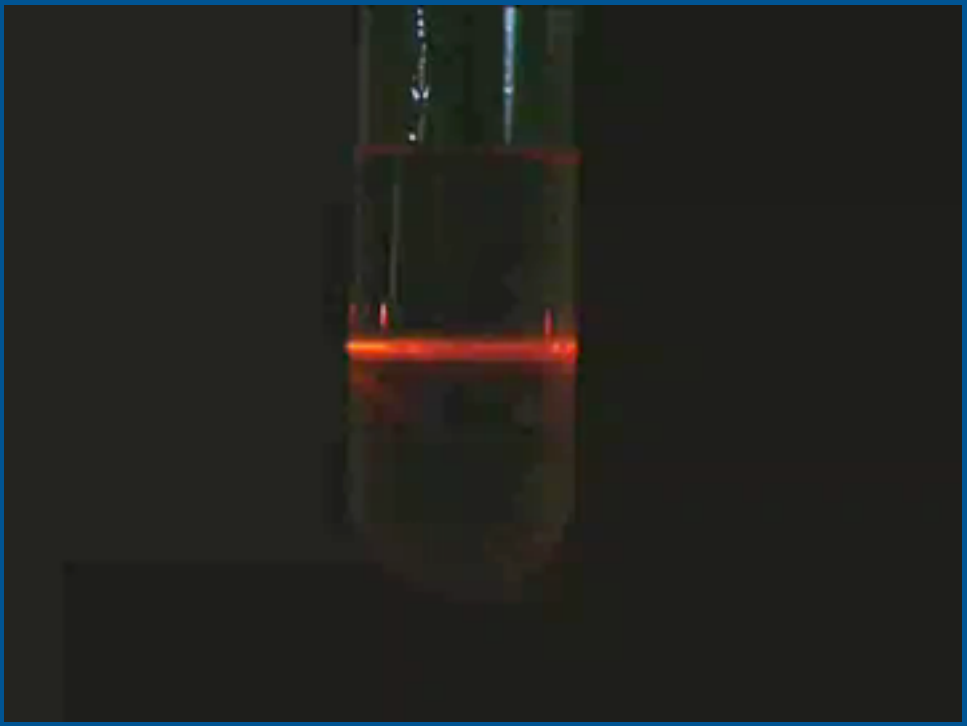
$$e^{n_{\text{steps}}(\beta - \alpha)}$$

Balance! (Power law)

α, β not balancing
above critical temperature

Critical Opalescence

Critical Opalescence



Critical Opalescence