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Research program

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Over one hundred years ago, Poincaré pioneered the concept of the qualitative study of ordinary differential equations and dynamical systems. The idea was, in a way, revolutionary — instead of studying nature by studying a particular equation, Poincaré's vision was to study nature by studying the geometric objects that were created by sets of time-dependent mappings from the function spaces that formed solutions to the equations used to model nature. With the formulation of this research program Poincaré gave birth to much of what has comprised the mathematical study of dynamical systems and ordinary differential equations for the last one hundred years. Though his framework, it was hoped that a geometric quantification of the equations that govern nature could be understood. This framework is, in a very fundamental way, interdisciplinary in perspective because the geometric structures are defined by the function spaces and particular geometric constraints that are largely independent of particular equations. Moreover, this framework, when applied to particular sets of equations, is independent of the interpretations of the equations imposed by the various scientific communities. However, because of the level of abstraction, this framework is also difficult to connect to science in a practical way if one remains solely in an analytical realm. This particular set of Poincaré's ideas, in many ways, remained rather removed from the mainstream scientific communities however, until the advent of the computer. Starting with scientists such as Lorenz in the 1960's, scientists began studying physically relevant mappings by solving them with computers. This pursuit has lead to many important examples of time varying systems of equations — dynamics that were difficult to discover from a purely analytical perspective. In the 1970's and early 1980's, the mathematics and computational communities enjoyed some limited cross-pollination — many mathematicians undertook the careful study of particular sets of equations such as the logistic map, the Lorenz equations and the Hénon map which were originally studied numerically by natural scientists. However, in the 1980's, computing power remained rather limited; qualitative studies of spaces of mappings remained out of reach in a computational framework. In the time since the 1980's, dynamics, as I see it, has split into two rather distinct communities, the mathematics community and the computational (scientific) community. Of course each of these communities is very large and diverse, so this is a very course distinction to make; however, as work in the field has expanded, these two worlds have been getting increasingly distant and have developed what appear to be disjointed and sometimes contrary views of dynamics. The roots of this problem often lie in differences language and in tools used for analysis. One of the goals of my work is to help to bring these two communities closer to each other. I (and my co-authors) try to use language from both fields to explain problems and their solutions. Moreover, the problems I am studying are directed towards closing the gaps between the mathematical and computational viewpoints in an effort to contribute to Poincaré's original vision.

My research program utilizes the continuing advances in computing resources and is fundamentally a qualitative, computational study of dynamical systems. This approach lies in between the abstract, pure math study of dynamical systems and the scientific-minded numerical studies of particular natural systems. The work is close to mathematics in regards to the problems being addressed and the feature of not focusing on a particular equation that has a specific physical analogy. However, often the experimental scientific strategy of problem-solving is employed via statistical studies and broad physical analogies. The approach is fundamentally of interdisciplinary character because all concrete, deterministic, time-dependent models have geometric structures carved out in time associated with them that can, in theory, be related to the abstract mathematical results. While the interpretations of the parameters and features of a mapping can vary from application to application, the geometry does not. My work focuses in particular on high-dimensional dynamics. The emphasis is motivated primarily by the outline of Poincaré, to achieve a geometric understanding of mechanisms that yield persistent types of dynamics, to gain an understanding of the required ingredients for ergodic-like dynamics to remain stable with perturbations. Moreover, achieving a geometric understanding of the transitions under parameter variation between dynamical types — between fixed points, periodic and quasi-periodic orbits, high-entropy hyper-chaos and low-entropy spatially extended systems — is often fundamental to understanding the stability mechanics of these dynamic types in and of themselves. These issues are approached in a siege style using whatever tools and methods work best for unwrapping and understanding the dynamics. Thus, these studies employ both analytical tools and criteria such as hyperbolicity, entropy and Lyapunov exponents, as well as tools that are traditionally used in scientific fields such as scaling laws. This allows for the study of classes of systems such as dissipative (not absolutely continuous) dynamical systems, that are very difficult to handle using only analytical tools. These methods often do not provide analytical arguments, but can speak to not only what kinds of dynamics exist, in much the same way that the mappings of Lorenz and Hénon did 30 years ago, but also to the sorts of assumptions that are reasonable (e.g. hyperbolicity) with respect to analytical arguments. The hope is that they can also aid in suggesting useful directions of analytical study. The computational and analytical problems that get addressed range from random matrix theory, bifurcation theory, time-delay dynamical systems, embedding theorems, functional analysis (as applied to neural networks), neural networks, Lyapunov exponents, dynamical stability theorems from dynamical systems, and scaling laws of all types. Finally, because the space of mappings used most commonly in my work are neural networks which form a very general function space and have many practical training algorithms associated with them, it is hoped that they can be used to work out a connection between the abstract and computational communities. Training ensembles of neural networks on the prototypical examples from the computational and mathematical communities can forge a bridge of understanding and put the work of these respective fields in context with each other. This is a step in the direction of linking Poincaré's abstract framework with the physical world he envisioned the framework resembling.

The framework

Most of my past and current work utilizes the space of feed-forward, scalar, time-delay, artificial neural networks. There are three reasons for using this space of neural networks. First, there is considerable theory supporting their generality by combining the time-delay embedding theorems of Takens and Sauer and Yorke and the neural network approximation theorems of Hornik et. al. Time-delay neural networks can approximate all C^r mappings and their derivatives (to arbitrary order) on compacta. Second, the space of neural networks with finitely many parameters can have a probability measure imposed on them. This means that they form a function space that can approximate the C^r function space but yet yield a manageable measure. Thus probabilities and frequencies can be discussed. The existence of a probability measure induces many invariance-to-measure type problems, but those issues are also present and are key problems in the more practical world of computational studies. Finally, this type of neural networks are often used for reconstructing dynamics from time-series data. They can be trained and there are significant practical tools available to form a bridge for comparison between the mathematics and computational communities.

High-dimensional, high-entropy versus high-dimensional, low-entropy dynamics.

One of the key differences between the mathematics and computation communities are the models they think of as prototypical. Most often, the computation community draws on the high-dimensional, low-entropy, spatially extended systems and coupled map lattices for their intuition. The high-dimensional, low-entropy cases have lots of intermittancy, synchronization, and resemble spatially extended physical systems. These systems can often be approximated with low-dimensional examples. Moreover, many computationalists use the logistic map as a prototypical example of low-dimensional chaos. Recently the mathematics community has shown that the logistic map has some very particular characteristics, such as dense stable periodic orbits, that are likely quite unique to this unimodal map. The mathematics community often uses examples such as Anosov diffeomorphisms, Smale horseshoes, or combinations of these or related diffeomorphisms which fall more into the high-dimensional, high-entropy setting. However, Anosov diffeomorphisms, for example, are mappings with determinant-one, and thus represent a non-dissipative scenario which is a very uncommon property in most natural systems. Coupled-map lattices and Anosov diffeomorphisms have very different properties, and this yields very different perspectives about what is common with respect to dynamics. The neural networks have both phenomena at various parameter settings. Thus, one goal is to use the neural networks to demonstrate that the geometric stability mechanisms for high-entropy versus low-entropy systems are very different. Quantifying and qualifying the high-entropy, high-dimensional geometrical mechanisms was begun in my thesis and several subsequent papers. Quantifying and qualifying the difference between the high-entropy and low-entropy systems is ongoing work.

Accomplished work: The high-entropy region of parameter space has been carefully studied. We have formulated various notions of stability and subsequent stability conjectures regarding a new geometric mechanism of stability with respect to a dynamics type. This work is discussed in the following papers:

Albers, D. J., Sprott, J. C., Crutchfield, J. P., "Persistent Chaos in High Dimensions" http://arxiv.org/abs/nlin.CD/0504040

Albers, D. J., Sprott, J. C., "Structural Stability and Hyperbolicity Violation in Large Dynamical Systems" http://arxiv.org/abs/nlin.CD/0408011

Dechert, W. D., Sprott, J. C., Albers, D. J., 1999. "On the Probability of Chaos in Large Dynamical Systems: A Monte Carlo Study," J. Econ. Dynamics and Control, 23 1197-1205

Albers, D. J., Sprott, J. C., Dechert, W. D., 1996. "Dynamical Behavior of Artificial Neural Networks with Random Weights," in Intelligent Engineering Systems Through Artificial Neural Networks, ed. by C. H. Dagli, M. Akay C. L. P. Chen, B. R. Fernandez, and J. Grosh, vol. 6 of Artificial Neural Networks in Engineering, pp. 17-22. ASME Press, New York

Current and future projects: I am currently work on, with various collaborators, a careful characterization of the low-entropy, high-dimensional, spatially extended region of parameter space with a particular focus on a geometric comparison with the high-entropy.

Hyperbolicity and center bunching

The structure of hyperbolicity and center bunching in high-dimensional, high-entropy systems is a very concrete connection between math and computation communities.

The concept of hyperbolicity is of utmost importance; many of the dynamics stability theorems as well as theories that prove the existence of Lyapunov exponents (e.g. Pesin theory) rest on various assumptions about hyperbolic structures. Studying hyperbolicity unlocks the geometry of the attractors in a global sense. The space of neural networks can approximate nearly any dynamical system. This characteristic affords the opportunity to study the nature of hyperbolicity for a diverse set of dynamics in a probabilistic (Monte Carlo) and practical manner. Moreover, it is possible to characterize the prevalence of hyperbolicity (non-hyperbolicity) and comment on how frequently, and in what dynamical circumstances, the various hyperbolicity assumptions that are necessary for various important theorems, are satisfied.

The concept of center bunching is an assumption regarding the "degree" of continuity a mapping must have for it to be ergodic (center bunching is specifically related to the exponent in the definition of Holder continuity). Center bunching of the derivative is a required assumption for the current proofs of the Pugh-Shub stable ergodicity theorem. The existence of such an assumption raises several important questions. First, in general, with respect to common computational frameworks, is center bunching required for ergodic-like dynamics? Second, how frequently is the center bunching criterion satisfied in practice? Finally, for what types of dynamical systems is center bunching relevant in a practical sense? Sometimes assumptions required for theorems have very fundamental "physical" reasons for existing; and sometimes they are required either because better arguments can't be found, or because the mathematics is not very relevant to the "real world" in a particular circumstance. My feeling and hope is that center bunching is "physically" relevant because it is related to the glue that is continuity, but we will have to wait and see what the data says.

Future and current projects: The study of local hyperbolicity variation along particular orbits is in the computation phase, meaning that the data is currently being collected from Beowulf clusters. The center bunching experiment is currently being constructed.

Scaling in the Lyapunov spectrum

My collaborators and I have been concerned with two problems regarding scalings with Lyapunov exponents: 1) the scaling between the individual exponents versus parameter variation, and 2) the scaling in dimension and number of parameters of a graph of the number of positive exponents versus parameter variation. The scaling between the individual exponents is something that has long concerned both physicists and mathematicians. One hope is that in our systems there is a renormalization that will develop, collapsing all of the positive exponents to a single exponent. Such a finding would aid greatly in explaining the geometric structure of the dynamical systems we study, under parameter variation. A scaling in the number of positive Lyapunov exponents that depends on the number of neurons, the number of dimensions, and one of the parameters (the variance of the weights of the neural networks) allows for an analysis of the asymptotic limit in the dimension that can address the topological variation under parameter variation.

Accomplished work: A careful study of scaling laws in the number of dimensions and parameters and quantities such as the maximum Kaplan-Yorke dimension, the maximum entropy, and the maximum number of positive exponents is complete and is summarized in:

Albers, D. J., Sprott, J. C., Crutchfield, J.P., "High-dimensional dynamics: scaling laws and general dynamics"

Future and current projects: The analysis of the scaling with respect to the number of parameters and the number of dimensions in the number of positive Lyapunov exponents versus parameter variation is being performed. The scaling between individual exponents is future work.

Transitions between fixed points and high-entropy dynamics

The transition from fixed points to chaos is often referred to as the route to chaos. Currently, for our mappings, we are concentrating on three particular directions with respect to this transition in high-dimensional dynamical systems. The first direction involves commenting on the most probable bifurcation from a fixed point, given our construction. The second direction is with regards to achieving a statistical understanding of the bifurcation sequences between the first bifurcation from a fixed point and the bifurcation to chaos. Leading up to the transition from non-chaotic to high-entropy dynamics, we have observed a decoupling cascade in the strong stable directions that is very similar to what is observed in the transition to chaos in Hamiltonian systems. Thus, the third direction is a systematic understanding of the geometric decoupling of the strong stable directions just before the onset of chaos.

Accomplished work: An understanding of the probability of a first bifurcation type as the dimension of the neural networks increases is given in the following papers:

Albers, D. J., Sprott, J. C., "Probability of local bifurcation type from a fixed point: A random matrix perspective" http://arxiv.org/abs/nlin.CD/0510060

Albers, D. J., Sprott, J. C., Dechert, W. D., 1998. "Routes to Chaos in Artificial Neural Networks with Random Weights," Int. J. Bifurcation and Chaos 8, 1463-1478

Work regarding the sequence of bifurcations between the first bifurcation from a fixed point and the bifurcation to chaos in high-dimensional neural networks can be found in:

Albers, D. J., Sprott, J. C., "Routes to chaos in high-dimensional dynamical systems: a qualitative numerical study" http://www.santafe.edu/ albers/research/papers/bif-seq.pdf

Future and current projects: In the paper entitled "Routes to chaos in high-dimensional dynamical systems: a qualitative numerical study," we observed the previously discussed phenomena of the decoupling of the stable directions before the onset of chaos. Understanding this phenomena in particular is a topic of future work. Analytical arguments regarding the probability of bifurcation type from a fixed point are being constructed.

Training the neural networks

Training the neural networks gets to the heart of the mathematics and computational science dilemma. Using the studies and tools discussed above, the geometry of the dynamics of neural networks can be characterized by their weight distributions. One can train ensembles of networks on classic dynamical systems, such as coupled logistic maps and Anosov diffeomorphisms, and then one can discuss dynamics as determined by weight distributions. Moreover, it is a method of putting the two classic examples in context with one another and comparing them.

Future and current projects: Training algorithms for the neural networks are currently be written, however, this project is still in the preliminary, development stages.

Looking forward: Future work

The research I intend to continue working on is targeted at the greater problem of linking abstract dynamical systems to the natural world — to push forward a geometric understanding of natural processes. The point of this is to explore the commonality and differences between natural systems without ignoring the specifics of these systems. To achieve these ends, one must be able to not only piece to together an understanding of scientific problems using mathematics, but also form an understanding of mathematics problems using science. Fundamentally, theorists in all fields who study time-evolving, deterministic systems in practice are studying dynamical systems. Thus, in the end, the computational results and the mathematics results must at least not be contradictory and hopefully will be unified into a single picture. However, there are large gaps between where analytical results leave off and where the computational results begin. For instance, the computational understanding of the Hénon map and the analytical understanding of the Hénon map are just beginning to converge after 20 years of work. Aside from connecting the computational with the analytical, work also needs to be done connecting the theoretical models with scientific results. This of course includes understanding when the frameworks utilized in theoretical investigations will not apply; understanding when tools work and when they do not is vital to understanding what the diagnostics are implying when used. This is fundamentally an interdisciplinary problem, for one must not only understand and have skills using tools provided by mathematics and computational science, but one must also have an understanding of fields addressing the measurable, scientific world, such as physics, economics, atmospheric science (Lorenz equations), space science (Hénon map), and the biological sciences (logistic equation). I feel that a major strength of my background and education is that I have worked with and have studied under a variety of academicians, including mathematicians, economists, experimental physicists, physical chemists, ecologists, and neuro-scientists who have provided me with an understanding of their respective problems and argument styles. My primary focus is the study of computational dynamical systems that are closely related to, and that interact with, abstract mathematics. However, the studies are always with goals set in understanding mathematics as related to natural processes. Thus, my research has thus involved researchers and influences from many branches of physical, social, mathematical, and computational science, and it will continue to have the flexibility to interact with a variety of scientific endeavors.