Aggregation of Labor Inputs in the Economics of Growth and Planning: Experiments with a Two-Level CES Function

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I. Introduction

Most recent attempts to explore the relationship between education and economic growth have been based, often implicitly, on a measure of the aggregate supply of labor services in the economy. A variety of such measures have been proposed, both in the study of the sources of economic growth, and in models for the efficient allocation of resources in the educational system.

The need for an aggregate measure of the supply of labor services arises when we seek to determine the historical growth contribution of changes in labor quantity and quality. Similarly, in the determination of the efficient allocation of resources in the educational system it is ordinarily helpful to have a measure of the aggregate supply of labor services in the economy; for it is by effecting increases in the total supply of labor services that the schooling system makes its primary contribution to future economic growth.

In both the study of the sources of growth and educational planning, the measure of the aggregate supply of labor services must be constructed for periods of time quite removed from the present. We ordinarily do not have adequate time-series data on the relative earnings of labor by schooling category which would be required to construct the labor supply index in a historical sources-of-growth analysis. Similarly, we have no information on the likely movement of relative earnings of various types

The material in this article draws heavily on sections of my book, *Planning Educational Systems for Economic Growth*. I am grateful to the Harvard University Press for kindly permitting me to publish the material here. I would like to acknowledge the competent assistance of Alice Crampin and Matthew Lambrinides, and the helpful comments of Finis Welch, Zvi Griliches, Christopher Sims, and a number of my colleagues in the Project for Quantitative Research in Economic Development. Portions of this research were supported by the Project for Quantitative Research in Economic Development through funds provided by the National Science Foundation and by the Agency for International Development under Contract CSD 1543.

of labor in the future, and thus are unable to construct the projected future measure of labor services necessary in the study of educational planning. In each case we must use current relative earnings in our labor aggregation function, or an estimate of the structure of wages in the past or the future. Denison, for example, used 1949 relative earnings to construct his measure of labor services in the U.S. economy over the period 1909–58. And in an earlier paper, I used contemporary relative earnings of labor with different levels of schooling in constructing the objective function of an intertemporal educational planning model.[1]

Of course, the assumption of constant relative earnings implied that the elasticity of substitution among labor inputs is infinite—for all practical purposes, labor may be considered a single (composite) factor. An alternative view of the relationship between education and growth is proposed in the so-called manpower requirements approach to educational planning. Here, the elasticity of substitution among labor factors (and between them and other factors) is taken to be zero; the labor requirements of the economy may be deduced from the projected pattern of output, without reference to the relative costs of labor.

In this paper I will present estimates of the elasticities of substitution among labor with different levels of schooling, and use these estimates to construct a two-level, constant elasticity of substitution function for the aggregation of labor inputs. I will then compare the measure of total labor services based on this estimated aggregation function with a number of other measures based on assumed values of the elasticity of substitution—ranging from one to infinity. The evidence presented here lends support to the view that elasticities of substitution among different labor inputs are high, and that for many purposes there may be no significant empirical loss in accepting as a working assumption the proposition that the elasticities of substitution among labor inputs are infinite.

I have in mind an economy with an aggregate production function

$$Y = h(K, L_1, \ldots, L_n), \tag{1}$$

where Y is a measure of annual output, K is a measure of the annual flow of capital services, and, $L_1, \ldots, L_n$ are the levels of input of labor, classified by n different levels of schooling. The question to which this paper is addressed is based on the assumption that it is legitimate to rewrite (1) as:

$$Y = H[K, f(L_1, \ldots, L_n)], \tag{2}$$

[1] See Denison (1962), Selowsky (1967), Bowles (1967), and Adelman (1966). The rate of return approach to educational planning is based on similar assumptions with respect to the validity of current relative earnings as a measure of future relative scarcities.

we then seek the proper specification of the function

$$L^* = f(L_1, \ldots, L_n) \tag{3}$$

where L^* is the total supply of (quality adjusted) labor services.

The concept of an aggregate supply of labor services is based on the assumption that the production function (1) is separable in the form of (2). Lack of data on the aggregate capital stock and other nonlabor inputs prevents my testing this crucial assumption for the countries in my sample. The requirements of separability are stringent, namely that the levels of nonlabor inputs have no bearing on the relative marginal productivities of the different types of labor inputs. Thus in choosing to measure the aggregate supply of labor services in the economy, I have obscured the possibility of differences in the degree of complementarity between nonlabor inputs and the various types of labor. Where the requirements of separability are not met, the concept of the aggregate supply of labor services may be a hindrance rather than an aid to clear thinking concerning the role of education in the growth process.[2]

II. The Elasticity of Substitution among Labor with Different Levels of Schooling

The analysis is confined to twelve economies, both rich and poor, on which there are data on mean earnings and quantities of male labor classified by years of schooling.

I would like to estimate the elasticity of substitution, σ_{ij}, between all types of labor, L_i and L_j. This concept represents the inverse of the percentage change in the ratio of wages associated with a given percentage change in the ratio of labor quantities. More explicitly:

$$\sigma_{ij} \equiv -\frac{d(L_i/L_j)/(L_i/L_j)}{d(w_i/w_j)/(w_i/w_j)}$$

$$\equiv -\frac{d\log(L_i/L_j)}{d\log(w_i/w_j)}.$$

As conventionally defined, the elasticity of substitution is a static concept and refers only to two factors in a given production process. We are considering here a case with many factors of production, only two of which are being observed explicitly in any single estimate. Moreover, the context is more aptly described as "historical" rather than static; other

[2] Throughout, we make the assumption that relative labor earnings measure relative marginal products. I will not deal here with the usual questions concerning the validity of this assumption, or the measurement of the economic effects of schooling by differences in earnings among workers classified by years of schooling. For a survey of some of these issues, see chapter 2 in Bowles, *Planning Educational Systems for Economic Growth.*

things are not equal. Specifically, the capital stock per worker and the sectoral composition of output vary greatly among the countries in our sample. Finally, our data refer not to a single production process, but to economy-wide aggregates. Thus our estimated elasticity is not a parameter of some underlying technological relationship or production function, for it will reflect the combined influence of: the partial elasticity of substitution in production between L_i and L_j; the degree of complementarity or substitutability between each of the two types of labor and excluded factors of production; and differences in both technology and the composition of final demand.

Of course, we would like to estimate a function which would tell us the effect of each of these influences (taken separately) on the relative marginal products of different types of labor. This function would allow the planner to predict changes in the relative marginal productivities of labor inputs on the basis of the other elements of his plan, such as the rate of capital accumulation, and changes in the composition of final demand. Although the following estimates of the relative importance of differences in the composition of final demand are a step in the right direction, we are still unable to provide a complete equation appropriate to the analysis of the role of education in growth.

Given this unsatisfactory situation, it is this historical concept of the elasticity, rather than the static one, which is more appropriate to the process of planning and the analysis of past growth. We would like to know the changes in relative marginal products of factors as the composition of the labor force becomes progressively better educated. The educational transformation of a labor force is a historical process in which other factors are not held constant, and which is ordinarily accompanied by substantial changes in the composition of final output. Thus, the elasticity estimated from our international cross section of countries at various stages of development is in fact the best available estimate.

The data allowed the identification of three classes of workers, those with zero to seven years of schooling, called L_1, those with eight to eleven years, L_2, and those with twelve or more years of schooling, L_3. The relationship among the data for two of the three pairs of labor are presented in figures 1 and 2.[3] The equations used to estimate each of the three elasticities of substitution are of the form:

$$\log\left(\frac{W_i}{W_j}\right)_k = \hat{a} + \hat{b}_{ij}\log\left(\frac{L_i}{L_j}\right)_k + u_k \qquad k = 1,\ldots, 12,$$

where the k subscripts refer to countries.

[3] A complete description of sources, and the raw data are presented in my *Planning Educational Systems for Economic Growth.*

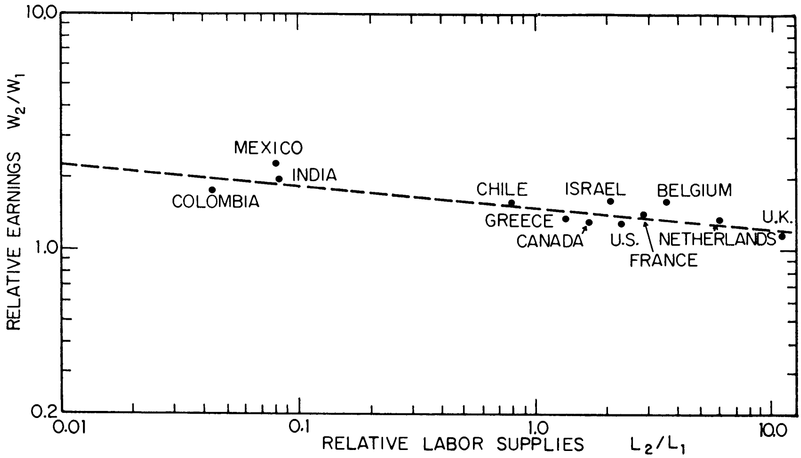

FIG. 1.—Relative earnings and relative labor supplies for labor with 8–11 and 0–7 years of schooling.

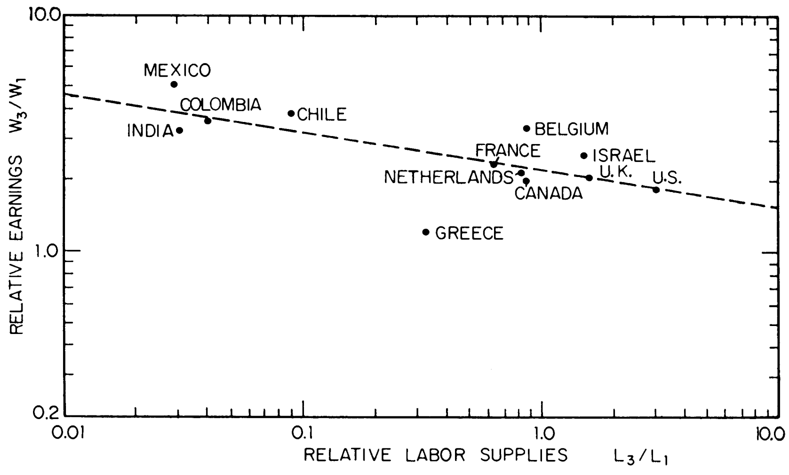

FIG. 2.—Relative earnings and relative labor supplies for labor with more than 11 and 0–7 years of schooling.

In each case the composition of the labor force has been taken as the exogenous variable. This is just the reverse of the usual assumption in the case of the firm which is assumed to adjust its hiring of factors to exogenously determined factor prices. However, given the fact that the educational composition of a population is determined to a significant degree by political, cultural, and other noneconomic considerations, it seems more reasonable to represent the relative factor prices as adjusting to a factor supply situation which is (largely) exogenous.[4]

The estimate of the elasticity of substitution is:

$$\sigma_{ij} = \frac{-d \log (L_i/L_j)}{d \log (w_i/w_j)} = \frac{1}{-b_{ij}}.$$

The estimated equations (with t-statistics in parentheses and related information) are:

$$\log \frac{w_1}{w_2} = -.4263 - \underset{(-4.54)}{.0831} \log \frac{L_1}{L_2} \tag{4}$$

$$R^2 = .6734 \qquad \sigma_{12} = 12.0,$$

$$\log \frac{w_1}{w_3} = -.8228 - \underset{(-3.13)}{.1552} \log \frac{L_1}{L_3} \tag{5}$$

$$R^2 = .4955 \qquad \sigma_{13} = 6.4,$$

$$\log \frac{w_2}{w_3} = -.5486 - \underset{(-.049)}{.0049} \log \frac{L_2}{L_3} \tag{6}$$

$$R^2 = .0002 \qquad \sigma_{23} = 202.0.$$

The first estimate implies that, when considered as part of a growth process (as represented by our cross section), a 12 percent change in the ratio of L_1 to L_2 is associated with a 1 percent change in the relative earnings. The other equations can be interpreted analogously.[5]

[4] Contemporary applications to schools are certainly not independent of the relative earnings of various types of labor. Given the apparent stability of relative earnings over time, the existing stocks of labor must be taken as endogenous to some extent. Thus here is a degree of simultaneity in the relationships. The assumption of exogenous labor supplies is not as innocuous as it might appear at first sight. If we assume that the relative costs of schooling, as well as the demographic characteristics of the population, are identical among countries, there is only one set of relative wages—common to all countries—which will achieve equilibrium in the human capital market. This is true regardless of the elasticities of substitution in the underlying production relations. Thus if our view of the world were the polar opposite of that adopted here, namely that the adjustment of labor quantities to a common marginal rate of transformation among labor categories achieves human capital market equilibrium at every moment, our estimates of the elasticity of substitution would tell us nothing about technology. We would be bound to find high but spurious estimates of the elasticity of substitution among labor inputs.

[5] If we exclude positive values of b_{ij} as implausible on a priori grounds, the expected value of the estimate no longer corresponds to the maximum likelihood estimate reported here. Were we estimating actual production relations (rather than relations dependent on the level of other nonmeasured inputs and variations in output composition, as well as technology), the exclusion of positive values would seem the correct procedure. Here the case is not so clear; there is a presumption against positive values, but they cannot be definitely excluded. In any case, the expected value of the estimate of b_{12} and b_{13} would be only very slightly larger in absolute value (and the estimated elasticities correspondingly lower) were we to exclude positive values of b_{ij}. On the other hand, exclusion of positive values of b_{23} yields an expected value of

The estimates are consistent with the hypothesis that there is no strong negative relationship between relative factor prices and relative factor quantities. Although in each case the relationship is, as expected, negative, in one case the relationship is very insignificant, and in all cases the estimated elasticities are high. All are significantly greater than three at the 99 percent level of significance.

The results suggest the possibility of aggregating the two best-educated categories of labor and thus allowing an estimate of the elasticity of substitution between two labor categories comprising the entire male labor force. The necessary condition for this aggregation is that the marginal rate of substitution between L_2 and L_3 be independent of the quantity of L_1, or that: $\partial(w_2/w_3)/\partial(L_1/\bar{L}) = 0$, where $\bar{L}$ is the total labor force (Leontief 1947*a*, 1947*b*). This hypothesis was tested directly by introducing $L_1/\bar{L}$ into equation (3) above. The hypothesis of no relation was accepted at conventional levels of significance.

A simple method of aggregation is suggested by the finding that the marginal rate of substitution between L_2 and L_3 is independent of the relative quantities of these two factors.[6] The two best-educated categories of labor can be aggregated to form a new, synthetic factor, L', where $L' = L_2 + L_3(w_3/w_2)$ and w_3/w_2 is the mean ratio of wages of L_3 to L_2 from the international cross section. The earnings of this factor, w', are defined as: $w' = (L_2w_2 + L_3w_3)/L'$.

The relationship between relative quantities and relative factor payments using this new aggregated factor, L', is estimated from:

$$\log \frac{w'}{w_1} = 1.3403 - \underset{(-5.165)}{.1242} \log \frac{L'}{L_1} \tag{7}$$

$$R_2 = .7274.$$

The implied elasticity of substitution between well-educated and less-educated labor is 8.

While the magnitude of these estimates seems to support strongly the view that elasticities of substitution among labor inputs are high, a number of reservations should be noted. First, we can expect that the presence of errors in the independent variables, as well as unemployment, has intro-

approximately $-.1$ (implying an elasticity of substitution of 10). This estimate is based on the fact that the maximum likelihood estimate of b_{23} is virtually zero. The expected value of a random normally distributed variable with mean zero and standard deviation, σ, over positive values is σ, which in this case is .1.

[6] It is possible to derive equation (6) from equations (4) and (5). The resulting equation bears only the slightest resemblance to that estimated. Thus the empirical basis for aggregating the most educated two categories of labor is somewhat weaker than the above discussion indicates. Nonetheless, given our estimates, there seem to be stronger grounds for this particular form of aggregation than for any other.

duced upward biases in the estimated elasticities of substitution. However, the magnitude of both biases appears to be small.[7]

A second reservation arises from the use of cross-section rather than time-series estimates. Because we intend to apply these results to problems of educational development within a given country over time, we must investigate the possibility that the high elasticity apparent from the cross-section estimates is not representative of actual historical development of any given economy. Although it was not possible to investigate the question adequately, the results of a very partial test are not inconsistent with the hypothesis that the actual historical elasticity in a given country is at least as high as the elasticity estimated from the cross section.[8]

The high estimated elasticity is in part due to the variability in the composition in final demand, largely through foreign trade, which allows countries to produce relatively more of those goods whose input structure is intensive in the factors which are abundant in that country.[9]

A measure of the impact of indirect factor substitution through changes in final demand can be derived from the reestimation of equation (7), this time taking explicit (though crude) account of the influence of the commodity composition of output through the introduction of a variable representing the percentage of national product originating in agriculture,

[7] First, in the presence of errors in the data earnings and labor quantities, the regression coefficient of the independent variable (the ratio of labor quantities) will be biased toward zero; the estimate of the elasticity of substitution is accordingly upward biased. An upper limit to this bias can be established by making the ratio of labor quantities the dependent variable. Recalling that there are errors in both variables, we may regard the estimated regression coefficient of the ratio of wages as a downward biased estimate of the elasticity of substitution itself. Reversing the order of the variables in equation (7), this procedure yielded a lower limit estimate for the elasticity of substitution between well-educated and less-educated labor of 5.9 with a standard error of 1.1.

Second, for all the countries in the sample, my labor input measures include unemployed workers. The existence of unemployment suggests that the observed wages and quantities do not represent a market equilibrium. Because this problem is particularly severe in the poor countries, and for the lowest schooling category of labor, the estimated elasticities are biased upward. In order to calculate plausible limits for this bias, I assumed that the equilibrium quantities of L_1 in each of the four poor countries (India, Columbia, Mexico, and Chile) is 80 percent of the observed inputs, or that the unemployment rate among the least-educated workers is 20 percent. The resulting estimate of the elasticity of substitution, from equation (7), is 7.9, or virtually the same as the original estimate.

[8] Our only available time series is for the United States and this is limited to five observations between 1939 and 1963. I estimated equations for these U.S. observations, and while significance levels were low, the implied elasticities were uniformly higher than those reported for the cross section. A test of the hypothesis that the time-series and cross-section data were drawn from the same population could not be rejected.

[9] The validity of the Heckscher-Ohlin model of international trade extended to include human capital as a factor of production is strongly supported by the research of Donald B. Keesing (1961) and K. Roskamp and G. McMeekin (1968).

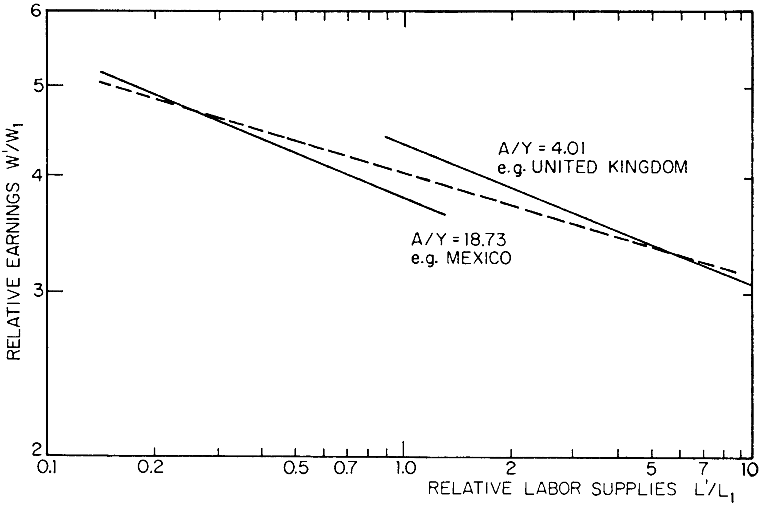

FIG. 3.—The effect of changes in the sectoral composition of output on the demand for educated labor. The dashed line represents equation (7); the solid lines are from equation (8); A/Y is the share of the agricultural sector in national income.

A/Y.[10] The resulting equation (with t-statistics in parentheses) is:

$$\log \frac{w'}{w_1} = 1.45 - \underset{(-5.38)}{.1704} \log \frac{L'}{L_1} - \underset{(-1.96)}{.0082} \frac{A}{Y} \tag{8}$$

$$R^2 = .8094 \qquad \sigma = 5.9.$$

The results, which are illustrated in figure 3, are consistent with the explanation suggested above. The effect of holding constant the share of national product originating in agriculture is to reduce the estimated elasticity of substitution from 8.0 to 5.9. Thus the ability of an economy to vary the composition of output seems to considerably enhance the overall substitutability among labor inputs in the economy.

III. An Empirical Evaluation of Alternative Measures of the Supply of Labor Services

In order to gain some measure of the empirical significance of the high elasticities of substitution estimated above, I will construct a labor aggregation function based on these estimates. The estimate of the total labor

[10] The figures for the percentage of output coming from agriculture were taken from the UN *Yearbook of National Accounts Statistics* (1961). The definition of national product used is somewhat inconsistent. Definition of the concepts used appears in the notes to the individual country tables in the *Yearbook*.

supply services and the average labor quality in each of our twelve countries, based on this function, will be compared with analogous estimates using two alternative assumptions about the elasticity of substitution among labor inputs.

First, we may consider a method of aggregating labor inputs recently used by Edward F. Denison and others in the study of the contribution of schooling to growth:

$$L^{\infty} = L_1 + L_2 w_2 + \dots + L_n w_n, \tag{9}$$

where w_i is the ratio of earnings of labor of type i to the earnings of labor, in the case of type 1, an arbitrarily chosen type. It is assumed that relative earnings reflect relative marginal products. When the function is based on a given year's relative earnings, and used to study changes in the supply of labor services over an extended period of time, this form of aggregation assumes an infinite elasticity of substitution between all pairs of labor; each marginal rate of substitution is unaffected by changes in the relative factor quantities.

Alternatively, we may assume an elasticity of substitution equal to one, and write:

$$L^1 = L_1^{a_1} L_2^{a_2} \dots L_n^{a_n}, \tag{10}$$

where $a_1, \dots, a_n$ represent the (constant) share of total wages earned by each of the n types of labor.

A further method of aggregation embodying less restrictive assumptions concerning the elasticity of substitution can be written:

$$L^* = (d_1 L_1^{-v} + d_2 L_2^{-v})^{-1/v}, \tag{11}$$

where d_1, d_2 and v are estimated parameters and $d_1 + d_2 = 1$. In this case the elasticity of substitution, σ, is constant, and equal to $1/(1 + v)$. In order to estimate this function we make use of the assumption that relative factor prices are equal to the relative marginal products of the factors, or that

$$\frac{w_1}{w_2} = \frac{d_1}{d_2}\left(\frac{L_1}{L_2}\right)^{-1/\sigma}. \tag{12}$$

Thus our data on relative earnings and quantities of labor of each type are sufficient to estimate the function. Because this measure incorporates an empirical estimate of the elasticity of substitution, I will use it as a criterion measure, with which to evaluate the quantitative biases involved in using the assumed elasticities underlying the other methods of aggregation. However, notice that this aggregation function refers to only two types of labor. It may be extended to include more, in the form

$$L^{**} = (d_1 L_1^{-v} + d_2 L_2^{-v} + \dots + d_n L_n^{-v})^{-1/v}, \tag{13}$$

where

$$\sum_{i=1}^{n} d_j = 1,$$

yet this form has the limitation that the partial elasticities of substitution between each pair of labor types are required to be equal.[11] The evidence from the regression equations (4), (5), and (6) suggests that this is not the case.[12] Thus I have adopted a two level procedure (Sato 1967) in which

$$L^* = f[L_1, g(L_2, L_3)], \tag{14}$$

and where the second aggregated factor is

$$L' = g(L_2, L_3) = (g_2 L_2^{-k} + g_3 L_3^{-k})^{-1/k}, \tag{15}$$

where: $g_2 + g_3 = 1$. However, since we found no relationship between the ratio L_2/L_3 and w_2/w_3 (see eq. [6]) we may assume that the relevant elasticity of substitution is infinite. In this case k equals -1, and the function (15) reduces to:

$$L' = g_2 L_2 + g_3 L_3,$$

which is exactly the form of aggregation used earlier in the paper. The coefficients g_2 and g_3 represent the mean relative earnings of the two types of labor.

Thus we arrive back at the first level equation, L^*, and using the notation of our international sample, we can now write:

$$L^* = (d_1 L_1^{-v} + d' L'^{-v})^{-1/v}. \tag{16}$$

Notice that our regression equation (7) provides the estimates of both the elasticity of substitution and the parameters d_1 and d'. Thus our final estimates for this aggregation function are:

$$L^* = [.7995\ (.3651\ L_2 + .6349\ L_3)^{.8806} + .2005\ L_1^{.8806}]^{1/.8806}. \tag{17}$$

Measurement of the parameters of the other aggregation functions was based on the data from the international cross section. The estimated values of each labor index are expressed as a fraction of the values for the United States in table 1. By dividing the aggregate labor input index by the for the United States. The degree of association between the various labor quality indices is represented by the appropriate zero-order correlation coefficients in table 3.

[11] This is the form suggested by H. Uzawa (1962).

[12] Because of the large standard errors of the estimate of the elasticity of substitution between labor of types two and three, the hypothesis that the magnitudes were in fact identical could not be rejected at conventional levels of significance. Nonetheless, the sizable differences in the estimated coefficients suggested that there would be some loss in assuming all elasticities to be the same.

TABLE 1

AMOUNT OF QUALITY ADJUSTED LABOR BY COUNTY ACCORDING TO VARIOUS LABOR AGGREGATION INDICES, EXPRESSED AS A FRACTION OF THE U.S. AMOUNT

	AGGREGATION INDICES			
COUNTRY	L (1)	L^* (2)	L^∞ (3)	L^1 (4)
United States	1.00000	1.0000	1.0000	1.0000
Belgium	.05215	.0447	.0417	.0534
Canada	.1031	.0867	.0848	.1182
Chile	.0400	.0233	.0231	.0299
United Kingdom	.3504	.2816	.2783	.2490
France	.2608	.2081	.2024	.2706
Greece	.0530	.0397	.0383	.0575
India	2.7822	1.4118	1.4524	1.265
Mexico	.1931	.0978	.1007	.0869
Netherlands	.0697	.0556	.0544	.0667
Colombia	.0884	.0446	.0462	.0345
Israel	.0104	.0096	.0095	.0113

NOTES.—Col. (1), L = total males economically active. Col. (2), method of calculation described in text. See equations (15)–(17). Col. (3), $L^\infty = L_1 + L_2(w_2/w_1) + L_3(w_3/w_1)$. The relative earnings are the means for the cross section of countries. Col. (4), $L^1 = L_1^{a_1} L_2^{a_2} L_3^{a_3}$. The wage shares are the mean wage shares for the sample of twelve countries.

numbers of workers, we have an index of labor quality. These indices are presented in table 2, expressed as a fraction of the value of the same index.

It is clear from table 3 that the measure using the assumption of an infinite elasticity of substitution among labor inputs (Q^∞) comes extremely

TABLE 2

QUANTITY OF LABOR PER WORKER: ALTERNATIVE MEASURES OF AVERAGE LABOR QUALITY AS FRACTION OF U.S. LABOR QUALITY

Country	Q^*	Q^∞	Q^1
United States	1.0000	1.0000	1.0000
Belgium	.8182	.7997	1.0230
Canada	.8427	.8226	1.1409
Chile	.5840	.5787	.7482
United Kingdom	.8035	.7942	.7105
France	.7980	.7761	1.0375
Greece	.7496	.7222	1.0844
India	.5075	.5220	.4547
Mexico	.5064	.5214	.4501
Netherlands	.7977	.7811	.8712
Colombia	.5042	.5228	.3899
Israel	.9239	.9138	1.0935

SOURCE.—Table 1.

NOTE.—Each measure, i, for any given country, j, is equal to $(L_j^i/L_j)/(L_{U.S.}^i/L_{U.S.})$. Thus $Q_j^o = (L_i^o/L_j)/(L_{U.S.}^o/L_{U.S.})$.

TABLE 3
ZERO-ORDER CORRELATION AMONG VARIOUS LABOR-QUALITY INDICES

	Q^*	Q^∞	Q^1
Q^*	1.000	.997	.854
Q^∞	.997	1.000	.820
Q^1	.854	.820	1.000

close to reflecting the quality of labor index as measured by the two-level function embodying the actual estimated elasticities. For all practical purposes, the two indices are identical when applied to this particular set of countries.[13]

IV. Conclusion

Although the empirical basis for this study is far from adequate (particularly the absence of capital stock data), the results presented here suggest that, given data constraints and the limits of computational feasibility, the assumption of constant relative earnings of labor in the study of both planning and growth is supported as a rough working generalization. The estimates cast serious doubt on the concept of an educational *bottleneck* as a barrier to economic growth in poor countries, for only with a low elasticity of substitution among labor inputs could such a bottleneck arise. (Of course, the absence of an educational bottleneck does not imply that educational resources are optimally allocated.)

In some cases it will be possible to use the labor aggregation function estimated here to project future movements in relative wages, or to estimate their movement in the recent past. While this is undoubtedly an improvement over the assumption of constant relative earnings, in all the experiments conducted thus far (on Colombia and Greece), the difference between the naïve assumption of constancy and the estimated movement in relative earnings based on actual changes in factor supplies has been minor (see Selowsky 1968; Bowles 1969).

[13] The infinite elasticity function, L^∞, was calculated using mean relative wages from the sample of twelve countries. The results would not have been different had we used either U.S. or Indian relative wages. The zero-order correlation among the L^∞ indices using different relative earnings are:

	L^∞ *mean*	L^∞ *U.S.*	L^∞ *Indian*
L^∞ mean . . .	.1.0	0.992	0.997
L^∞ U.S. . . .	...	1.0	0.980
L^∞ Indian . . .	...	...	1.0

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