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Evolutionary bargaining with intentional idiosyncratic play[☆]

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ABSTRACT

We study equilibrium selection in stochastic evolutionary bargaining games in which idiosyncratic play is intentional instead of random. In contract games, the stochastically stable state selected by intentional idiosyncratic play is the Nash bargain, rather than the usual Kalai-Smorodinsky solution.

1. Introduction

We extend the Binmore–Samuelson–Young (Binmore et al., 2003) approach to equilibrium selection in contract games and related bargaining games by imposing empirically plausible restrictions on the process generating idiosyncratic (non-best-response) play. (By contract game, Young (1998) means an asymmetric pure coordination game played by randomly matched players from two populations.) Our modification to the standard dynamic(Kandori et al., 1993; Young, 1993a) is motivated by our belief that agents who act idiosyncratically in economic conflicts are behaving intentionally, and thus do not "accidentally" experiment with contracts under which they would do worse, should the contract be generally adopted. We have in mind such idiosyncratic play as refusing to exchange under the terms of a contract that awards most of the joint surplus to the other party (for example locking out overly demanding employees). Like Bergin and Lipman (1996), who conclude that "models or criteria to determine 'reasonable' mutation processes should be a focus of research in this area", and Van Damme and Weibull (2002), our idiosyncratic play is state-dependent. But while these authors make error *rates* state dependent, we make the *distribution* of idiosyncratic play across the strategy space state-dependent, as in Bowles (2004).

The resulting dynamic based on intentional idiosyncratic play provides a more plausible account of historical real world transitions between economically important conventions, such as customary crop shares or the de facto recognition of collective bargaining by businesses. First, when non-best-response play is intentional transitions between contracts are induced only by the idiosyncratic play of those who stand to benefit from the switch, in contrast to the standard (unintentional) approach. Second, as one would expect, in the intentional dynamic where population sizes and idiosyncratic play rates differ, the population whose interests are favored is that whose members who engage in more frequent idiosyncratic play and who are less numerous.

We find that the contracts that are selected as stochastically stable under the intentional idiosyncratic play dynamic differ from those selected under the standard dynamic. Our dynamic selects the convention that implements the Nash bargain, while the standard dynamic selects the Kalai-Smorodinsky bargain (Young, 1998; Kalai and Smorodinsky, 1975). The difference is illustrated in the example in Table 1. The Kalai-Smorodinsky bargaining solution equates the

☆ Appendices may be found in the online version of the paper, available at: http://www.tuvalu.santafe.edu/snaidu/papers/sub_text_0506102.pdf.

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Table 1
Example 1.

Contract	0	1	2
0	5.60	0.0	0.0
1	0.0	12.20	0.0
2	0.0	0.0	36.1

ratio of the payoffs to the ratio of the players' best possible payoff, and thus is the contract pair $(1,1)$, as $12/20=36/60$. In contrast, the Nash solution is $(0,0)$, since the Nash solution is that which maximizes the product of the payoffs and $5\times60>12\times20>36\times1$.

In Section 2 we introduce intentional idiosyncratic play, present the main proposition of the paper, and characterize the stochastically stable state under intentional dynamics for a variety of cases.

2. The model

2.1. Setup

We consider two populations of sizes N and M, denoted R and C for row and column, playing an asymmetric bargaining game. Both row and column players have K strategies, with payoff functions given by $\pi^R(i,i)=a_i,\pi^C(i,i)=f(a_i)$ where $i\in S=\{1,2,...,K\}$ and f is positive and decreasing. We order the strategies such that $a_i<a_j$ for $i<j$, so the row player favors contracts with higher indices, and the column player favors contracts with lower indices. The off-diagonal payoffs are given by $\pi^R(i,j)=\pi^C(i,j)=0$ if $i>j$ and $\pi^R(i,j)=\lambda a_i,\pi^C(i,j)=\lambda f(a_j)$ if $i<j$, where $0\le\lambda\le1$. That is, agents receive some fraction of their demands if the demands together do not exhaust the surplus, and receive 0 otherwise. The contract game (Young, 1998) corresponds to $\lambda=0$ and the Nash demand game corresponds to $\lambda=1$. Clearly, the diagonal of the game matrix constitutes the set of pure Nash equilibria, and they are all strict and Pareto-optimal.

The dynamic is a familiar myopic best-response dynamic with inertia (Kandori et al., 1993; Binmore et al., 2003). The state of the dynamic is described by distributions of strategies in populations denote by (x,y) (x and y for row and column). Each period, all players are matched to play the contract game. Each time they are matched, agents revise their strategy with some probability and play the strategy they played last period with the remaining probability. To specify strategy revisions, first we denote by $BR_R(y)$ (or $BR_C(x)$) the best responses of the row player (or the column player, respectively) — the pure strategy which maximizes the expected payoff, $\sum_j\pi_R(i,j)y_j$ (or $\sum_j\pi_C(i,j)x_j$).

We model idiosyncratic behaviors by using multinomial random variables. Specifically, for the row population each period we draw a multinomial random variable $Z^R=(Z_0^R,Z_1^R,\cdots,Z_K^R)$ with parameters N and τ^R and suppose that Z_0^R agents play the best responses and the remaining $Z_1^R,Z_2^R,\cdots$, and Z_K^R players idiosyncratically choose strategy $1,2,\cdots$, and K. Here the probability vector τ^R consists of the probability, τ_0^R, with which each agent in the row population plays the best responses and the probabilities, $\tau_1^R,\cdots$, and τ_K^R, with which each agent play the idiosyncratic strategy $1,2,\cdots$, and K. Hence the probability vector τ^R and its support, that we will define shortly, play a key role to specify idiosyncratic behaviors of populations. We use a similar multinomial random variable Z^C to capture the idiosyncratic plays of the column population. With this updating rule, the dynamic yields a well-defined Markov chain (X_t,Y_t); we provide an example of such dynamics in the online Appendix for an illustration.

Given a strategy b, we note that $\{i:b<i\le K\}$ is the set of strategies that row population prefers to b because of the indexing of strategies, so the set $\{i:b<i\le K\}$ can provide a set of strategies from which an intentional idiosyncratic player draws. From this observation, we set

$$\tau_i^R(b)=\begin{cases}0 & \text{if } 1\le i<b\\ \dfrac{\epsilon}{K-b+1} & \text{if } b<i\le K\end{cases},\quad \tau_j^C(b)=\begin{cases}\dfrac{\epsilon}{b} & \text{if } 1\le j\le b\\ 0 & \text{if } b<j\le K\end{cases},$$

$$\tau_0^R(b)=1-\sum_{i=1}^{K}\tau_i^R(b),\quad \tau_0^C(b)=1-\sum_{j=1}^{K}\tau_j^C(b),\quad \tau^R=(\tau_0^R,\cdots,\tau_K^R),\ \text{and}$$

$\tau^C=(\tau_0^C,\cdots,\tau_K^C)$. Note that in an unperturbed process in which no idiosyncratic behavior exists, the set $\{i:b\le i\le K\}$ is empty for all b; no agent plays an idiosyncratic strategy. In an unintentional idiosyncratic process, agents choose idiosyncratic strategy from the whole strategy set; i.e., $\{i:b\le i\le K\}=S$ for all b, which means the support of error always equals S. The intentional idiosyncratic play distribution is state-dependent; for example, row only experiments with strategies that would give the idiosyncratic player a higher payoff when played as part of a pure strategy Nash equilibrium of the unperturbed game, so that column is best-responding with an offer that exhausts the surplus. This observation leads to the following definition. We write $Z\sim\mathcal{MN}(N,\tau)$ if Z follows a multinomial variable with N draws and a probability vector τ.

Definition 1.

- $(X_t,Y_t)_{t\in Z_+}$ is an *unperturbed* process if $Z_t^R\sim\mathcal{MN}(N,\tau^R(K+1))$ and $Z_t^C\sim\mathcal{MN}(M,\tau^C(0))$
- $(X_t,Y_t)_{t\in Z_+}$ is an *U-process* if $Z_t^R\sim\mathcal{MN}(N,\tau^R(0))$ and $Z_t^C\sim\mathcal{MN}(M,\tau^C(K+1))$
- $(X_t,Y_t)_{t\in Z_+}$ is an *I-process* if $Z_t^R\sim\mathcal{MN}(N,\tau^R(BR_R(Y_t)))$ and $Z_t^C)\sim\mathcal{MN}(M,\tau^C(BR_C(X_t)))$.

Clearly both the U-process and I-process are finite state space Markov chains and that the transition probability matrix of U-process is irreducible and aperiodic, so the chain admits a unique stationary distribution $\mu(\varepsilon)$. We are interested in the stochastically stable states namely, those that have positive weight in the limit of the distribution $\mu(\varepsilon)$ when $\varepsilon\to0$ following Young (1993a). We show that I-process is irreducible and aperiodic in the online Appendix.

2.2. Unintentional vs intentional idiosyncratic dynamics

The U-process is the standard mutation dynamics encountered in the literature (Kandori et al., 1993; Young, 1993a). Analyzing the I-process is the contribution of this paper. Binmore et al. (2003) show that the stochastically stable state in the U-process is the Kalai-Smorodinsky solution in the contract game, and the Nash bargaining solution in the Nash demand game. It is also useful to describe the transitions between states in the U-process; in the contract game they are driven by mistakes in the population who loses from the transition. Our I-dynamic, in contrast, has agents only erring in the direction that could benefit them if sufficiently many others did the same; thus the populations driving transitions are the ones that stand to gain. This difference in the relevant population mutations drives the differences in the stochastically stable state that the two processes select.

First, it is easily seen that each contract i is an absorbing state in the unperturbed process, where we identify the state where all agents in both row and column population play the same strategy i with contract i. Then following Binmore et al. (2003) we compute the resistance R_{ij}— minimum number of idiosyncratic players to move from the state i to the state j- in I-process, ignoring integer considerations:

$$R_{ij}=\begin{cases}N\dfrac{f(a_i)-\lambda f(a_j)}{f(a_i)+(1-\lambda)f(a_j)} & \text{if } i<j\\ M\dfrac{a_i-\lambda a_j}{a_i+(1-\lambda)a_j} & \text{if } i>j\end{cases}.$$

We call trees with R_{ij} edge resistances *I-trees*. From Theorem 1 in (Young, 1993a), we know that the I-stable state is contained in the root of the minimal I-tree. In the online Appendix A we show that the U-stable state in example 1 is the Kalai-Smorodinsky solution, while stochastically stable state under the I dynamic is the Nash solution. This is a general difference, as illustrated by the next proposition where we set $a_N^* = \underset{s \in [0,1]}{\arg\max}\, s f(s)$, and $a_G^* = \underset{s \in [0,1]}{\arg\max}\, s^M (f(s))^N$. We suppose that f is concave and normalize f in a way that $f(0)=1$ and $f(1)=0$.

Proposition 1. *Suppose the $a_i = i\delta$ and $i \in \left\{1, \ldots \frac{1-\delta}{\delta}, \frac{1}{\delta}\right\}$ for $\delta > 0$. Then we have*

(i) If $\lambda \leq 1$, a unique stochastically stable contract in the I-dynamic i^* exists, and is increasing in N/M
(ii) If $\lambda = 1$ and δ is sufficiently small, the stochastically stable contract i^* in the I-dynamic approaches $(a_G^*, f(a_G^*))$.
(iii) If $M = N$ and δ is sufficiently small, the stochastically stable contract i^* in the I-dynamic approaches $(a_N^*, f(a_N^*))$.

Proof. See Appendix B available online.

Note that if $\lambda = 1$ (the Nash demand game) the I- and U- dynamics select the same outcome (Young, 1993b). If $N = M$ then the symmetric Nash bargain solution is I-stable. Note also that if $\lambda \leq 1$ and N is not equal to M, the stochastically stable contract will be closer to the best contract for the group with lower population-size. Smaller groups are favored because the realized level of idiosyncratic play is more likely to exceed the critical level to induce a transition, and in the I dynamic groups benefit from the transitions which their idiosyncratic play induces.

Thus we find that a natural and empirically motivated restriction on idiosyncratic play in bargaining games may select different outcomes, as well as generating an empirically plausible transition dynamic in which smaller group size is an advantage, and groups whose idiosyncratic players induce transitions benefit as a result. For example, $N = M$ and $\lambda = 0$ (Contract game). Then the U-dynamic selects the Kalai-Smorodinsky solution (Young, 1998; Binmore et al., 2003), and the I-dynamic selects the Nash solution. Our I-dynamic is thus another class of bargaining interactions in which a standard result of axiomatic cooperative game theory is replicated by the non-cooperative play of only minimally forward looking individuals with limited information. The contrast between the I-dynamic and the standard model for the contract game illustrates the economic intuitions underlying these results. The key differences result from the fact that in the former transitions are induced by the idiosyncratic play of those who stand to benefit. In the U-dynamic the opposite is the case because it will always take fewer idiosyncratic players in one population to induce best responders in the other to shift to a contract that they prefer over the status quo than to induce them to concede to a less advantageous contract. In the U-dynamic, the deviations of one population induce the other population to coordinate on a contract that they strictly prefer to the status quo; while in the I-dynamic deviations by one population must induce the other population to coordinate on a strictly inferior contract.

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