The physical limits of communication *or*
Why any sufficiently advanced technology is indistinguishable from noise

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It has been well known since the pioneering work of Claude Shannon in the 1940s that a message transmitted with optimal efficiency over a channel of limited bandwidth is indistinguishable from random noise to a receiver who is unfamiliar with the language in which the message is written. We derive some similar results about electromagnetic transmissions. In particular, we show that if electromagnetic radiation is used as a transmission medium, the most information-efficient format for a given message is indistinguishable from blackbody radiation. The characteristic temperature of the radiation is set by the amount of energy used to make the transmission. If information is not encoded in the direction of the radiation, but only in its timing, energy, and polarization, then the most efficient format has the form of a one-dimensional blackbody spectrum.

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I. INTRODUCTION

Shannon's information theory[1,2] considers the set $\{x_i\}$ of all possible messages x_i that can be transmitted across a channel linking a sender to a receiver. In the simplest case the channel is considered to be free of noise, so that each message is received just as it was sent. Shannon demonstrated that the only consistent definition of the average amount of information carried by such a channel per message sent is

$$S = -\sum_i p_i \log p_i, \tag{1}$$

where p_i is the probability of transmission of the message x_i. We will take "log" to mean the natural logarithm as is common in statistical mechanics. In information theory base 2 logarithms are the norm, but this choice just multiplies S by a constant. In statistical mechanics, S is multiplied by the Boltzmann constant k; we will measure temperature in units of energy, so that $k=1$.

If there are no other constraints, then S is maximized when all the p_i are equal, that is, when all messages are transmitted with equal probability. If we transmit a constant stream of information in this way by sending many messages one after another, chosen with equal probability from the set $\{x_i\}$, then the resulting flow of data will appear completely random to anyone who does not know the language in which it is written; that is, it will appear to be white noise.

We now consider the corresponding problem for a message sent using electromagnetic radiation. In outline our argument is as follows. We assume that the sender of a radio message has a certain amount of energy available, and we ask what is the maximum amount of information that can be sent with that energy. Generically, this problem is one of maximizing the Shannon information, Eq. (1), for an ensemble of bosons (photons in this case) subject to the constraint of given average energy. The solution is familiar from statistical physics, because the formula for the Shannon information is identical to that for the thermodynamic entropy of an ensemble. The maximization of entropy for an ensemble of bosons gives rise to Bose–Einstein statistics and, for the case of electromagnetic waves, to blackbody radiation. By an exact analogy we now show that the most information-rich electromagnetic transmission has a spectrum indistinguishable from blackbody radiation.

II. THE TRANSMISSION OF INFORMATION USING ELECTROMAGNETIC RADIATION

In order to apply Shannon's theory to electromagnetic radiation, we must pose our problem in a form resembling the transmission of information over a channel. To do that, consider the following thought experiment. Suppose we have a closed container or cavity with perfectly reflecting walls, which, for reasons that will become clear shortly, we take to be a long tube of constant cross-sectional area A_t and length ℓ. Suppose also that we have the technical wherewithal to set up within this cavity any electromagnetic microstate, that is, any superposition of the modes of the cavity. Because the walls are perfectly reflecting, this microstate will remain in place indefinitely.

We consider using the cavity as a communication device in the following (slightly odd) way. Each possible message x_i that we might wish to send is agreed to correspond to one microstate, and we simply pass the entire cavity to the receiver of the message. The information transferred by the electromagnetic radiation stored within it is then given by

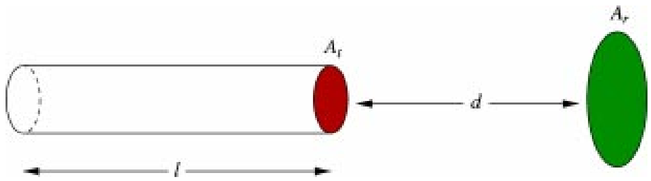

Fig. 1. The setup of our thought experiment. A tubular cavity of cross-section A_t (or more precisely its open end) is used as a transmitter. The message is received by a receiver of area A_r a distance d away.

Shannon's formula with p_i the probability that the cavity is in microstate i. This form of communication is not the same as sending a radio message, but as we will demonstrate, it has the same information content.

Now suppose that, as in the case of the simple channel, we wish to transmit a steady stream of information by exchanging a series of such cavities. We assume that the power available to make the transmission is limited—there is a fixed average energy "budget" $\langle E \rangle$ that we can spend per message sent—and we ask what is the greatest amount of information that can be transmitted per message. This amount is given by maximizing Eq. (1) subject to the constraint of fixed average energy but allowing any number of photons to be present in the cavity. The solution of this maximization problem is well-known[3,4] and gives the grand canonical ensemble in which

$$p_i = \frac{\exp[-\beta(E_i - \mu N_i)]}{Z}, \tag{2}$$

where E_i is the energy in microstate i, N_i is the number of photons, Z is the grand canonical partition function, and β and μ are Lagrange multipliers, usually referred to as the (inverse) temperature and chemical potential, respectively. If we now denote the microstates by the numbers of photons $\{n_k\}$ in each single-particle state k of the cavity, then it is straightforward to show that the average of n_k over many messages follows the Bose–Einstein distribution

$$\langle n_k \rangle = \frac{1}{e^{\beta \varepsilon_k} - 1}, \tag{3}$$

where ε_k is the energy of a photon in state k. We have set $\mu = 0$ because there is no chemical potential for photons in a vacuum.

Suppose now that, instead of handing over the entire cavity, we remove one of its ends and allow the photons to escape in the form of a radio transmission. We should envisage the end of the cavity as being the transmitter, not the cavity itself. The cross-section A_t of the cavity is the area of the transmitter, hence the subscript t. Usually we will want to direct the message toward a specific receiver. For concreteness we will assume that the receiver is a distance d from the end of the cavity and presents some finite area A_r to the cavity as depicted in Fig. 1. Only those photons in the cavity that have their momentum within the correct interval of solid angle will strike the receiver, possibly after reflecting one or more times off the walls of the cavity. (We use only the forward interval of solid angle, disallowing reflections off the rear wall of the cavity. If we allow such reflections, the transmission will convey twice as much information but last twice as long, making the rate of information transfer identical.) Given that our cavity has volume $V = \ell A_t$, the density of single-particle states satisfying this criterion, taking into account both polarizations of the photons, is $\rho(\varepsilon) = 2\ell A_t A_r \varepsilon^2 / d^2 h^3 c^3$, where h is Planck's constant and c is the speed of light. The power spectrum of our message—the average energy per unit time per unit interval of energy—is thus

$$I(\varepsilon) = \frac{2\ell A_t A_r}{d^2 h^3 c^3} \frac{\varepsilon^3}{e^{\beta\varepsilon} - 1}. \tag{4}$$

This functional form for the intensity $I(\varepsilon)$ is usually referred to as a blackbody spectrum. A blackbody spectrum is produced by a perfect thermal radiator in equilibrium at temperature $T = \beta^{-1}$, and, to a good approximation, by most astronomical bodies.

There is a small correction to Eq. (4) because photons may be delayed slightly if they have to take a path that reflects off the walls of the cavity. This correction goes inversely as the cosine of the angle subtended by the receiver at the transmitter. We have assumed that the distance between transmitter and receiver is large enough that this cosine is well approximated by 1.

Our transmission necessarily contains all the information needed to reconstruct the original microstate of the cavity, and therefore contains the same amount of information as that microstate. Furthermore, no ensemble of messages can exist that originates within a volume less than or equal to that of our cavity and conveys more information per message for the same average energy. If such an ensemble existed, reversing time would allow us to trap messages drawn from that ensemble within a cavity of the same size and create an ensemble of microstates that also had greater information content. This state of affairs would lead to a contradiction, because the blackbody spectrum maximizes the entropy.

Now we send a stream of information in the form of many such messages, each one resulting from one microstate of the cavity. Because of the tubular shape we have chosen for our cavity, each message will last a time ℓ/c and the transmission will have constant average intensity. The apparent temperature of the transmission is fixed by the size of the energy budget. Calculating the average energy $\langle E \rangle$ per message by integrating Eq. (4) over the energy ε and dividing by ℓ/c, we find that for a transmission with power P (the energy budget per unit time) the apparent temperature $T = \beta^{-1}$ is given by

$$T^4 = \frac{15 h^3 c^2}{2\pi^4} \frac{d^2}{A_t A_r} P. \tag{5}$$

The information transmitted per unit time, $\mathrm{d}S/\mathrm{d}t$, can be calculated using $S = \log Z - \beta\, \partial \log Z/\partial\beta$, noting that $\partial \log Z/\partial\beta = \langle E \rangle$. The result is

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \frac{8\pi^4}{45 h^3 c^2} \frac{A_t A_r}{d^2} T^3 = \left[\frac{512\pi^4}{1215 h^3 c^2} \frac{A_t A_r}{d^2} P^3 \right]^{1/4}. \tag{6}$$

Equation (6) gives the greatest possible rate at which information can be transferred by any electromagnetic transmission for a given average power P. It depends only on fundamental constants, the areas of the transmitter and receiver, the distance between them, and the average power, or equivalently, the apparent temperature.

Results similar to ours have been derived using different methods in Ref. 5 for information flow through a waveguide and in Ref. 6. These results differ from ours in that $A_t A_r / d^2$ is replaced by the waveguide's cross-sectional area.

For a transmitter and receiver of one square meter each, a meter apart, with a power of $P=1$ W, the information rate is 1.61×10^{21} bits per second. Note that the information rate increases as $P^{3/4}$, slightly slower than linear. It also increases with the area of the transmitter and receiver, so that the best information rate for a given energy budget is achieved for large antennas and low apparent temperature.

III. ONE-DIMENSIONAL TRANSMISSIONS

To reconstruct the microstate of the original cavity and hence extract the information from a transmission of this kind, we need to have complete information about the photons arriving at our receiver. Thus in addition to energy and polarization information, we need to know the transverse components of the momentum (or position) of each photon. In practice, we can only detect the transverse components with finite resolution, which places an upper limit on the energy range over which Eq. (4) is valid. Conversely, the finite size of the receiver places an upper limit on the wavelength of the photons that can be detected, and thus a lower limit on their energy. Therefore, for a given receiver, the most information-efficient spectrum will maximize the entropy conditioned on the frequency being between these limits.

In fact, most receivers are not capable of measuring the transverse components at all. Usually we have information about only the energy and timing of the arriving photons, and possibly their polarization. In this case Eq. (3) still holds, but the density of states becomes $\rho(\varepsilon)=2\ell/hc$, and Eq. (4) becomes

$$I(\varepsilon)=\frac{2\ell}{hc}\frac{\varepsilon}{e^{\beta\varepsilon}-1}. \tag{7}$$

Equation (7) is the form that the spectrum of a black body would take in a one-dimensional world, and we refer to it as a "one-dimensional blackbody spectrum." The apparent temperature $T=\beta^{-1}$ is then given by

$$T^2=\frac{3h}{\pi^2}P, \tag{8}$$

and the information transmitted per unit time is

$$\frac{\mathrm{d}S}{\mathrm{d}t}=\frac{2\pi^2}{3h}T=\left[\frac{4\pi^2}{3h}P\right]^{1/2}. \tag{9}$$

For an energy budget of 1 W, Eq. (9) gives a maximum information rate of 2.03×10^{17} bits per second. This bound also applies to broadcast transmission for which we do not know the location or size of the receiver and therefore cannot guarantee that it will receive all, or even most, of the photons transmitted.

Equation (9) has a long history, which has been well reviewed by Bekenstein and Schiffer[7] and Caves and Drummond.[8] It often is written for a single-polarization channel, in which case $\mathrm{d}S/\mathrm{d}t$ is $\sqrt{2}$ smaller. It was first derived by Lebedev and Levitin,[9] who considered the thermodynamics of the receiver, modeling it as a set of harmonic oscillators. Pendry[10] followed an approach similar to ours in which the field is in thermodynamic equilibrium.

As Yuen and Ozawa[11] have pointed out, these derivations implicitly assume that what Caves and Drummond[8] call a "number-state channel," in which information is received by counting photons with each frequency and polarization, is optimal. They prove this assumption using bounds on the quantum entropy. Finally, it is interesting to note, as in Ref. 7, that if we cannot measure transverse momenta, then the transmission rate of Eq. (9) is independent of c, the speed of the particles carrying the information. Thus phonons, say, are just as efficient as photons.

Interestingly, Eq. (9) also can be derived by a thought experiment in which we transmit the signal into a black hole and consider its increase in entropy. This derivation is closely related to the fact that Hawking radiation also obeys a one-dimensional blackbody spectrum.[12]

IV. DISCUSSION AND CONCLUSIONS

We have shown that optimizing the information efficiency of a transmission that employs electromagnetic radiation as the information carrier, with a fixed energy budget per unit time, gives rise to a spectrum of intensities identical to that of blackbody radiation. In fact, to an observer who is not familiar with the encoding scheme used, an optimally efficient message of this type would be entirely indistinguishable from naturally occurring blackbody radiation. One might be tempted to say that perhaps one could tell the two apart by spotting patterns within particular frequency bands of the message, or by performing some other decomposition of the signal. This possibility is however ruled out by the maximization of the Shannon information; any regularities that would allow one to draw such a distinction are necessarily the result of less-than-optimal encoding. For the case where the transverse momentum of photons is not used to encode information, or for broadcast transmissions, the spectrum is that of a one-dimensional black body. We also have shown that the characteristic temperature of the message is simply related to the energy used to send it, and we have derived an upper limit on the rate at which information can be transmitted in both cases.

We end with some discussion and speculations. First, we should point out that the ideas outlined here constitute only a thought experiment. By a sequence of deductions we have placed an upper limit on the information efficiency of electromagnetic transmissions, but we have not shown how to achieve that upper limit. Second, we note that reasoning similar to that presented here applies to transmissions using other radiative media as well. Because many natural processes maximize the Gibbs–Boltzmann entropy, they should give rise to spectra indistinguishable from optimally efficient transmissions. For instance, if we had a transmitter that could emit any type of particle (rather than just photons), it seems plausible that the optimal spectrum of particle types and energies would be that of Hawking black-hole radiation.[13]

Finally, we note that, in a recent science fiction novel,[14] the cosmic background radiation (which is roughly blackbody at 3 K) is revealed to consist of the highly compressed transmissions of long-dead alien civilizations.

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