CS361 Pretest

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This is a take-home pretest to see how well-versed you are in the math you'll need for this course. It will be graded! But it is also intended as a taste of the kind of reasoning we'll be doing. If you don't see how to answer a question, feel free to ask a friend for help. Due in class on Tuesday, August 31st.

1. If I multiply two n-digit numbers, how many digits does their product have?
2. Roughly speaking, what is $\log_2 1{,}000{,}000$? Answer without a calculator!
3. True or false: $a^{\log b} = b^{\log a}$.
4. Name a function f such that $f(x) = f(x-1) + 2$.
5. Name a function f such that $f(x) = 2f(x-1)$.
6. What happens the to the previous two questions if I specify that $f(0) = C$?
7. Name a function f such that $f(x) = f(x/2) + 1$.
8. Name a function f such that $f(x) = 4f(x/2)$.
9. Name a function f such that $f(x) = 2f(x/4)$.
10. What happens to the previous three questions if I specify that $f(1) = C$?
11. There are 10 popular vacation spots, and three people each choose one randomly without talking with each other. What is the probability that a) they all choose the same place? b) two of them choose the same place and the third is elsewhere? c) they choose three different places? (Note that these probabilities should sum to 1.)
12. What is $\sum_{i=0}^{\infty} 4^{-i} = 1 + 1/4 + 1/16 + 1/64 + \cdots$?
13. What is $\sum_{i=1}^{n} i = 1 + 2 + 3 + \cdots + n$?
14. Roughly (meaning up to a constant) what is $\sum_{i=1}^{n} i^k$, where $k > 0$ is some constant and n is large?
15. Roughly (meaning up to a constant) what is $\sum_{i=1}^{n} r^i$, where $r > 1$ is some constant and n is large?
16. Find the pattern in these numbers and give a formula for $f(n)$. Check your formula by plugging in several values of n. Hint: try looking at the differences or ratios between successive values, or subtract or divide the sequence by one that looks close to it. Note that the values of n are different in the two problems.

a)

n	0	1	2	3	4	5
$f(n)$	3	4	7	12	19	28

b)

n	1	2	4	8	16	32
$f(n)$	0	2	8	24	64	160

17. Suppose I have a balanced binary tree of depth d. How many leaves does it have? How many internal nodes does it have? When d is large, what fraction of the nodes are leaves?
18. Which of the following algorithms and data structures could you code in a language of your choice?

Bubblesort Mergesort Quicksort Stack Queue Linked List Binary Tree Heap