CS500 Homework Zero

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This is a sort of pretest to see how well-versed you are in the math you'll need for this course. It will not be graded! But it is intended as a taste of the kind of math we'll need. As always, collaboration is allowed if you get stuck, but please use this to assess your readiness for the course.

1. How many subsets does a set of size n have?

2. If I add two n-digit numbers, how many digits does their product have? What if I multiply them?

3. Roughly speaking, what is $\log_2 1{,}000{,}000{,}000$? Answer without a calculator.

4. True or false: $a^{\log b} = b^{\log a}$.

5. Name a function f such that $f(x) = 2f(x - 1)$.

6. Name a function f such that $f(x) = f(x/2) + 1$.

7. Name a function f such that $f(x) = 2f(x/4)$.

8. There are 10 popular vacation spots, and three people each choose one randomly without talking with each other. What is the probability that a) they all choose the same place? b) two of them choose the same place and the third is elsewhere? c) they choose three different places? (Note that these probabilities should sum to 1.)

9. Approximately, what is $\sum_{i=1}^{i} 1/i = 1 + 1/2 + 1/3 + \cdots + 1/n$?

10. Give a good approximation to e^x when x is close to zero. (Don't just say 1, please.)

11. True or false: if $f(n) = O(g(n))$, then $2^{f(n)} = O(2^{g(n)})$. Give a proof or a counterexample.

12. Suppose I have a balanced binary tree of depth d. How many leaves does it have? How many internal nodes does it have? When d is large, what fraction of the nodes are leaves?

13. NoC Problem 1.2 (the Pigeonhole Principle)

14. NoC Problem 1.7 (Hamiltonian grids)

15. NoC Exercise 2.4 (doubling the speed of the computer)