SURFACE PHYSICS OF TOPOLOGICAL INSULATORS

MASSLESS ELECTRONS, MASSIVE IONS

MICHAEL EL-BATANOUNY

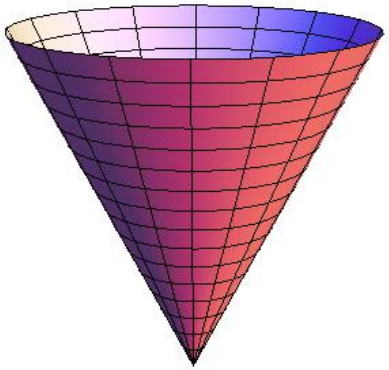
MASSLESS versus MASSIVE

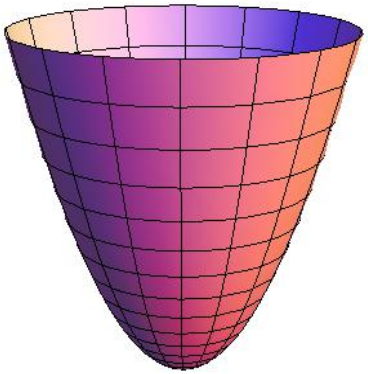
Newton's second law

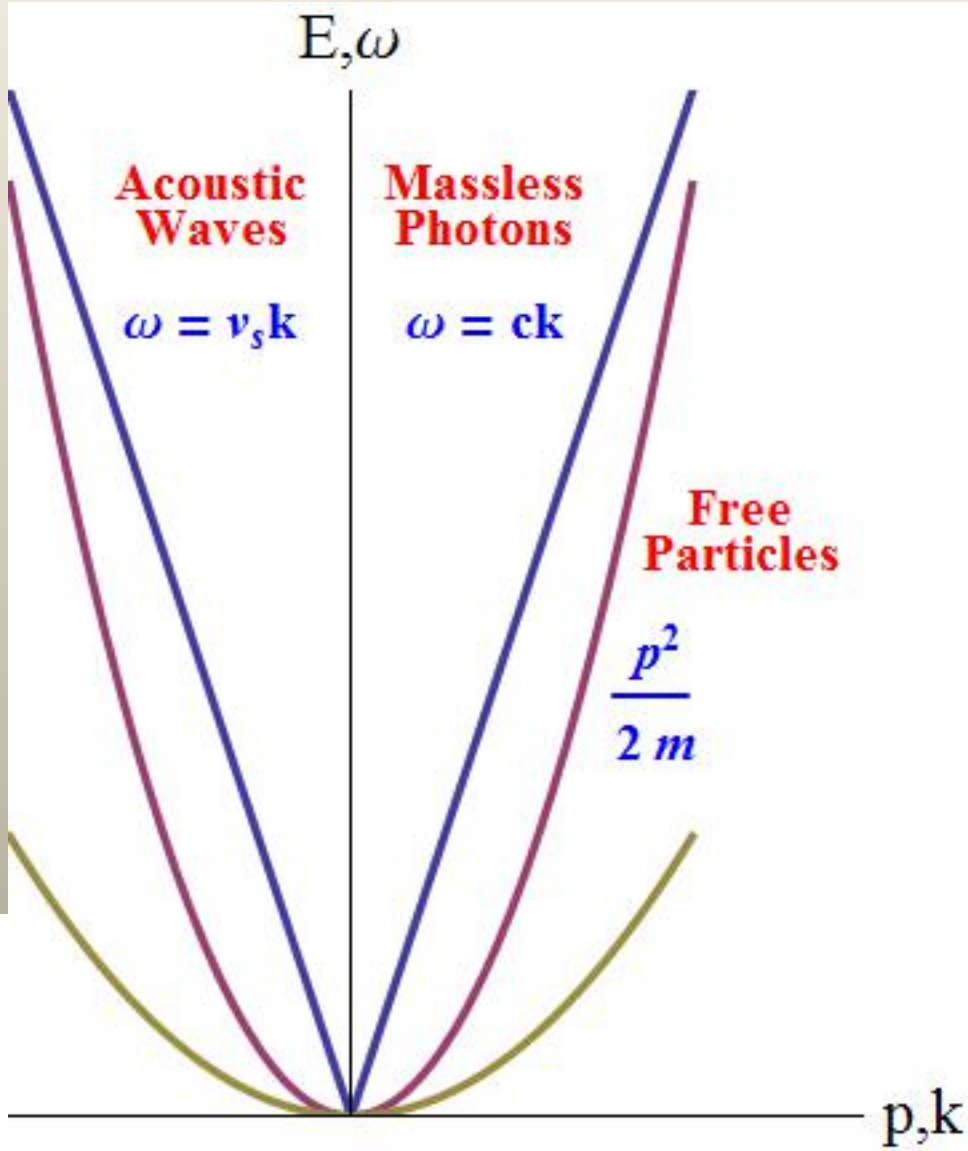
F = m a

= 0 a !

E= $\hbar\omega = c\hbar k$







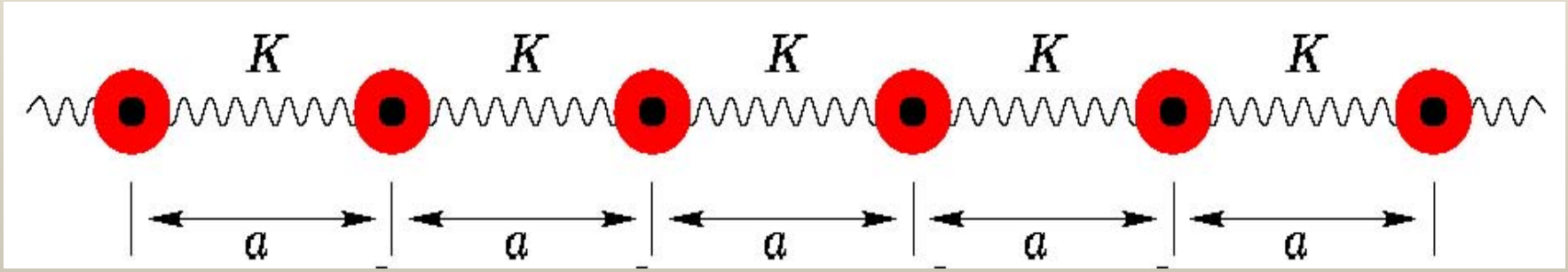
DIPERSION RELATION

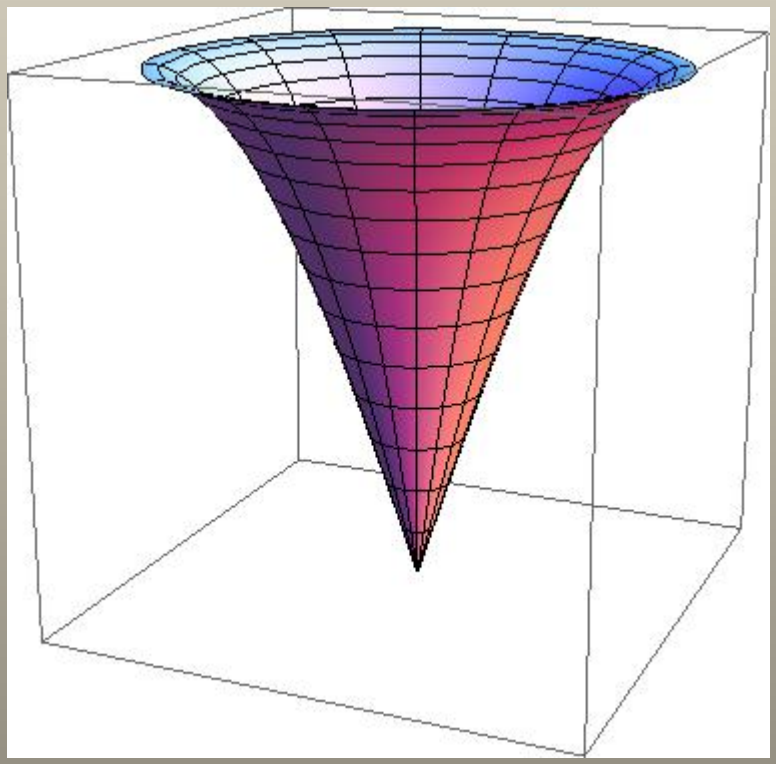
SPECIAL FEATURES
OF
DISPERSION RELATIONS
OF
ELECTRONS AND IONS
IN
SOLIDS

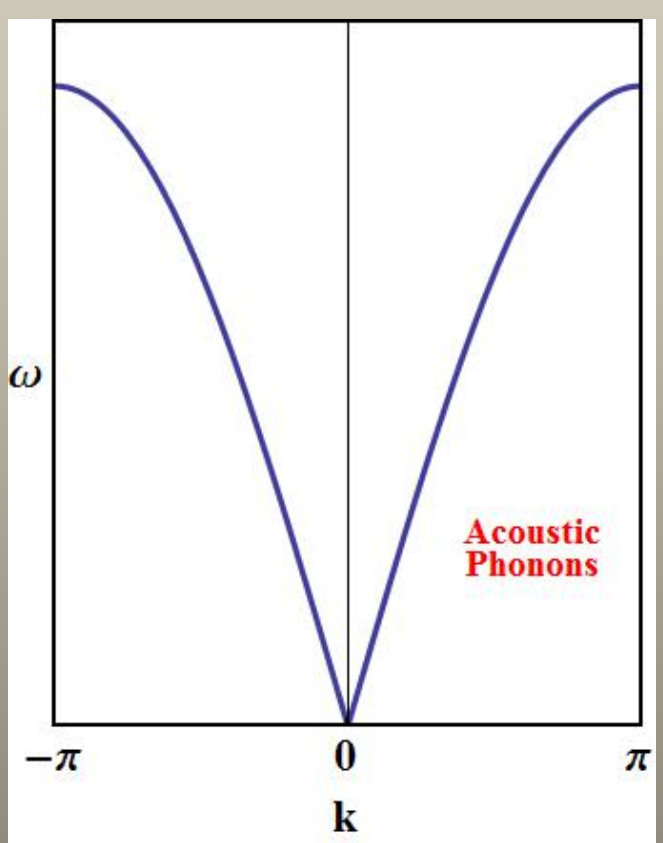
MOTION OF ATOMS (IONS)

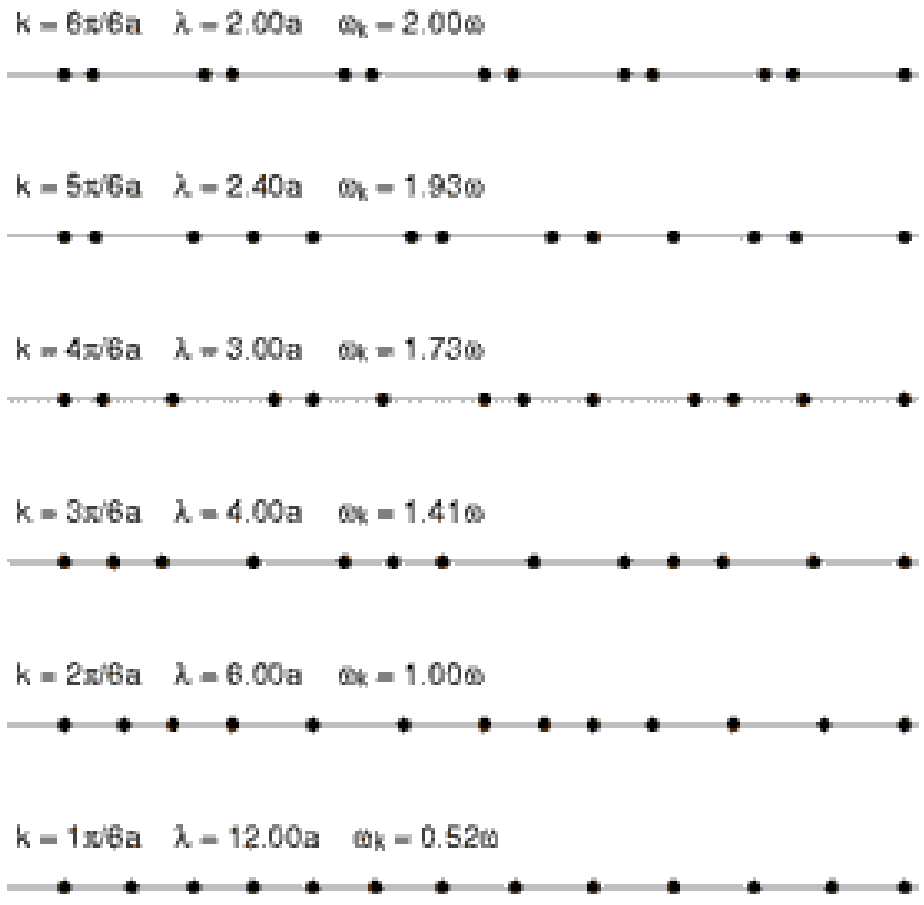
PARTICLE CHAIN

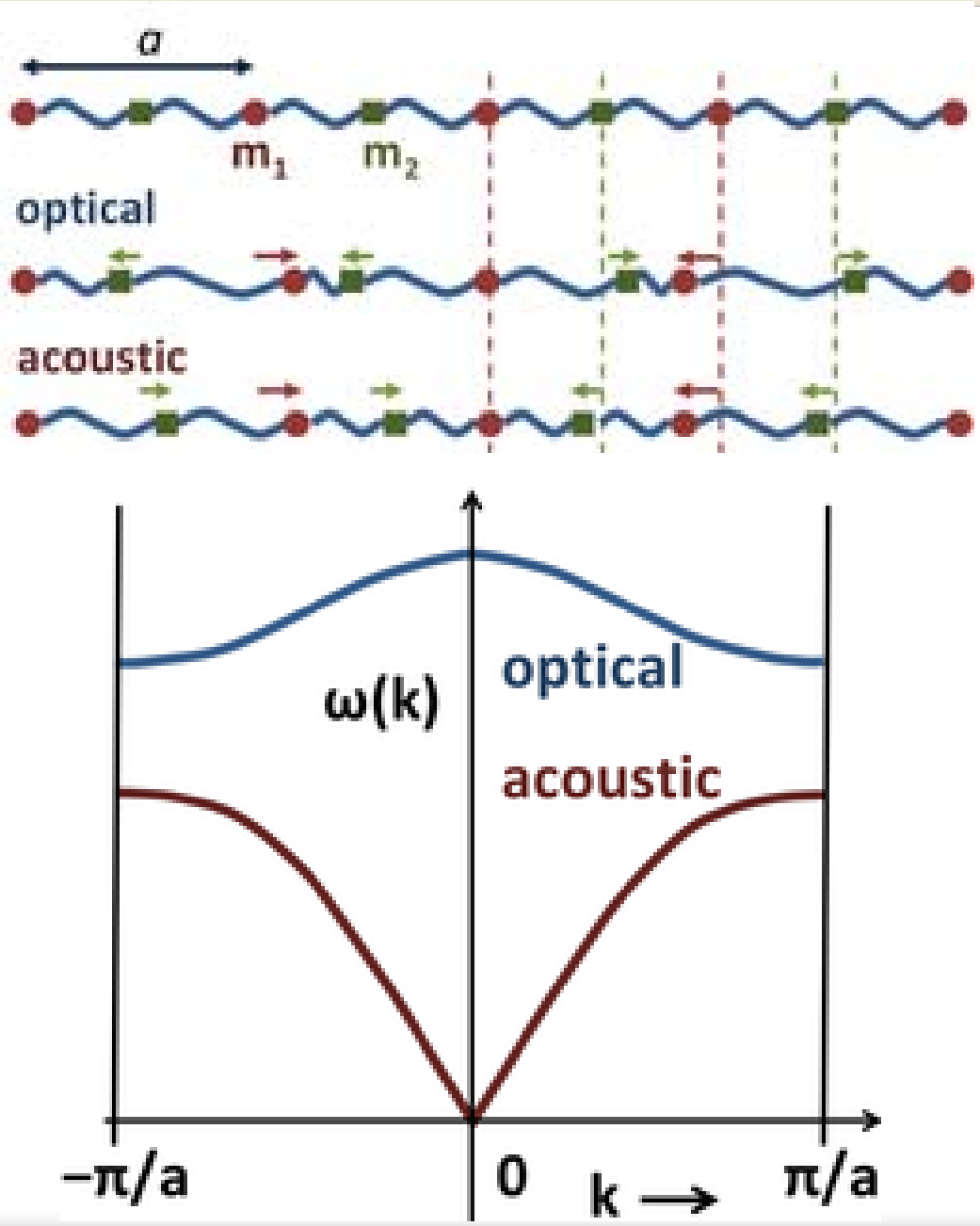
PHONONS



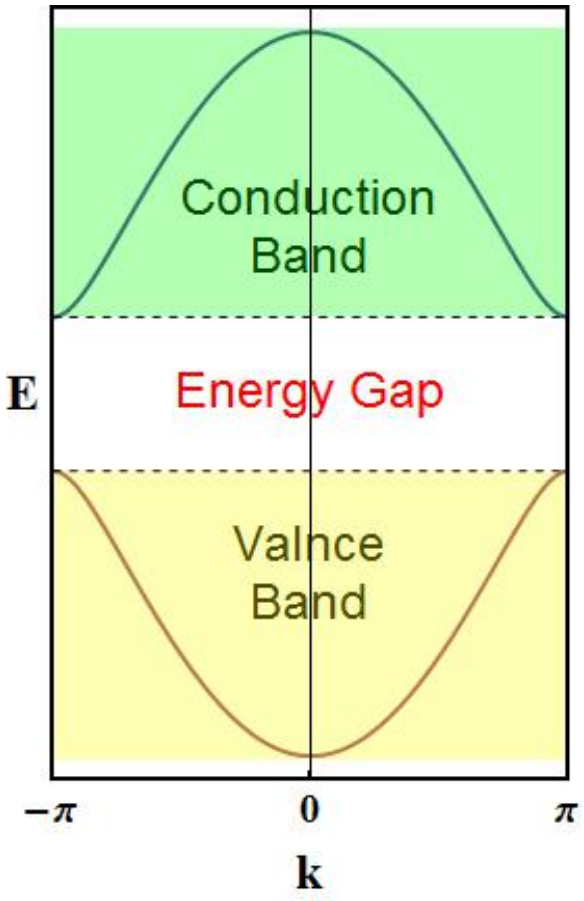




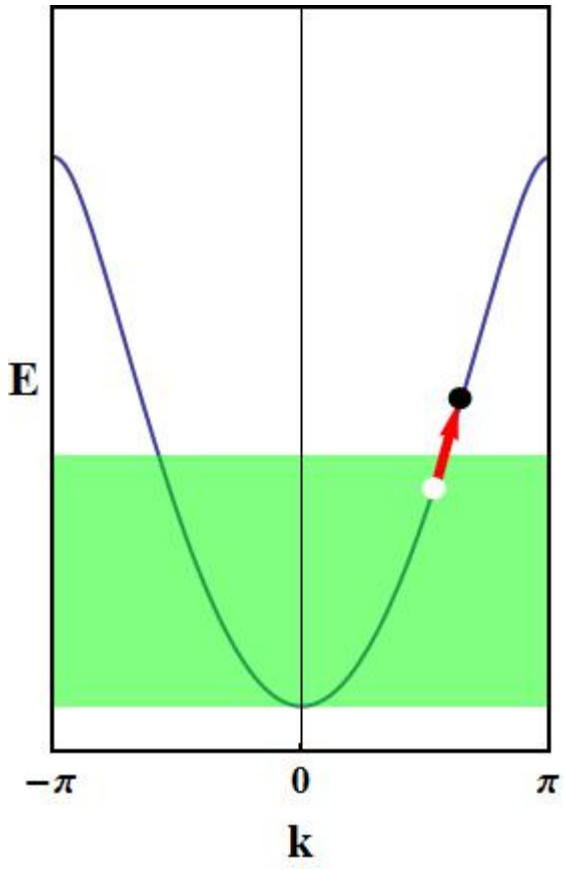

k = 6π/6a λ = 2.00a ω_k = 2.00ω
k = 5π/6a λ = 2.40a ω_k = 1.93ω
k = 4π/6a λ = 3.00a ω_k = 1.73ω
k = 3π/6a λ = 4.00a ω_k = 1.41ω
k = 2π/6a λ = 6.00a ω_k = 1.00ω
k = 1π/6a λ = 12.00a ω_k = 0.52ω


a
m1
m2
optical
acoustic
ω(k)
optical
acoustic
−π/a
0
k
π/a

MOTION OF ELECTRONS

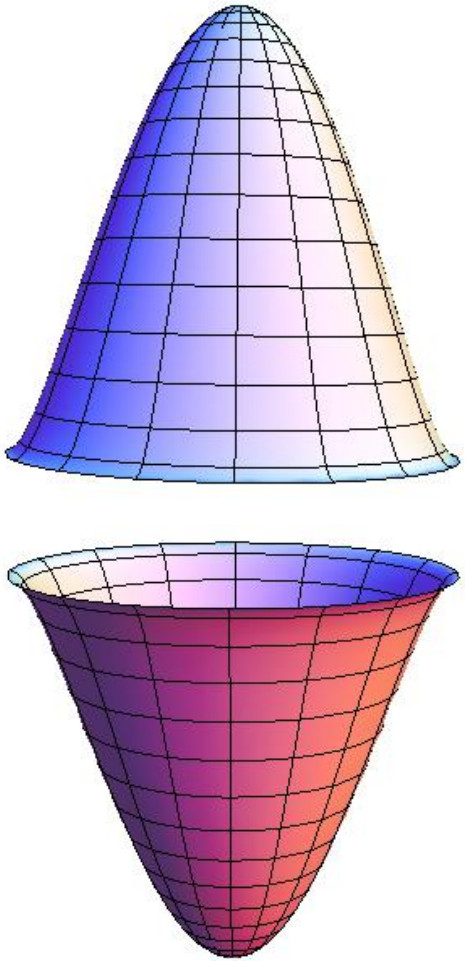


INSULATOR

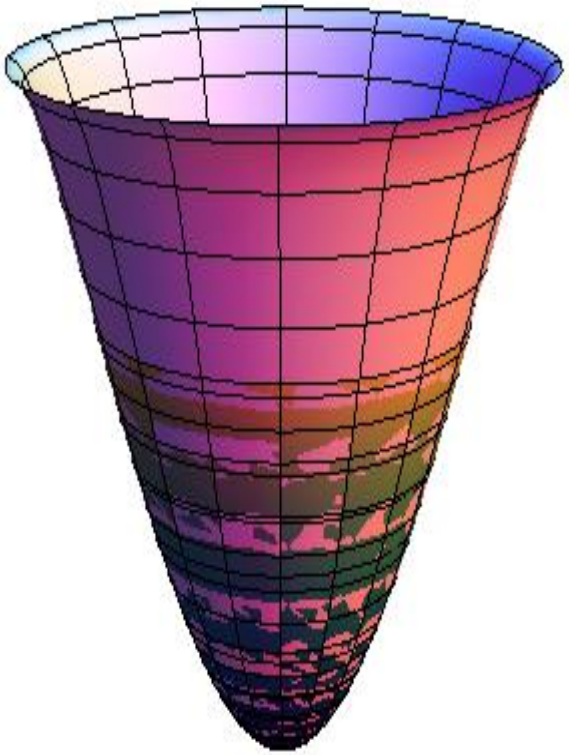


METAL

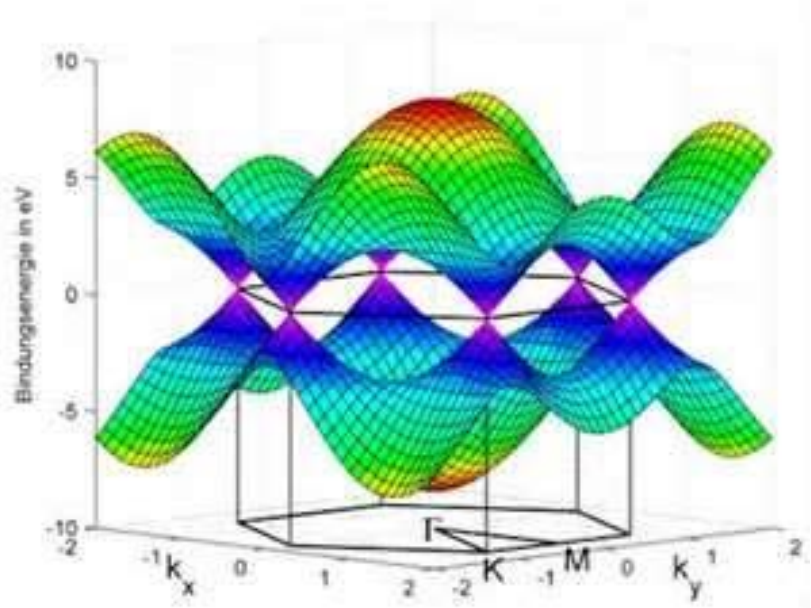
INSULATOR

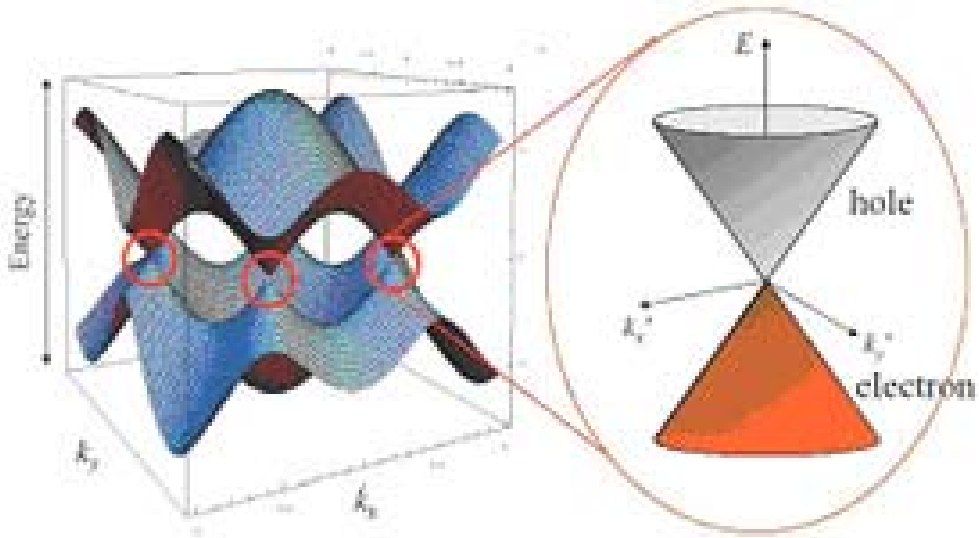


METAL

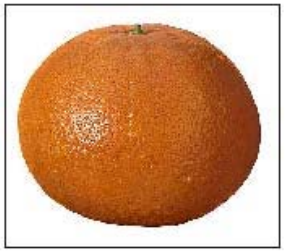


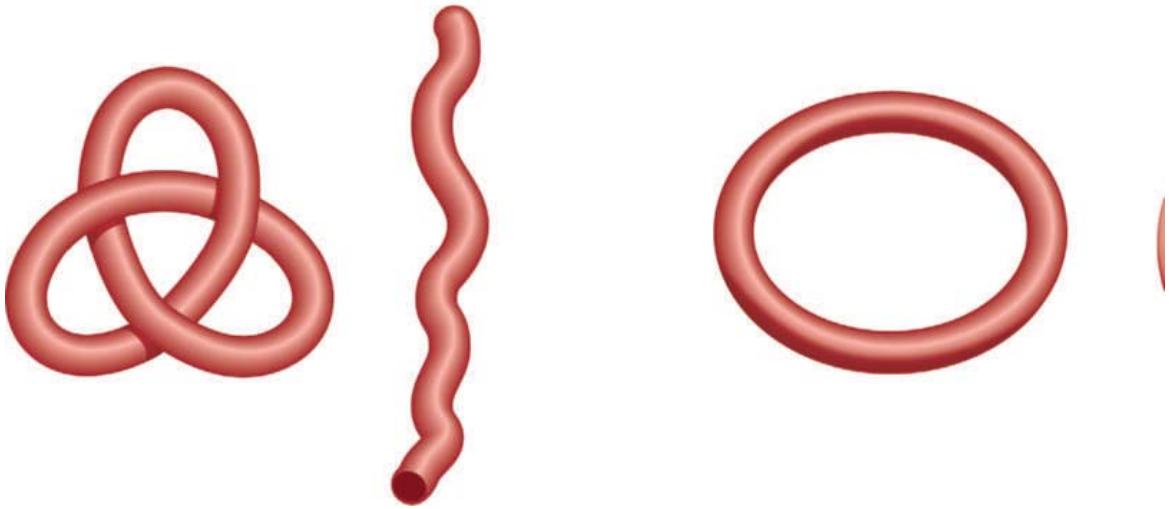
ELECTRONIC DISPERSION IN GRAPHENE





TOPOLOGICAL PROPERTIES

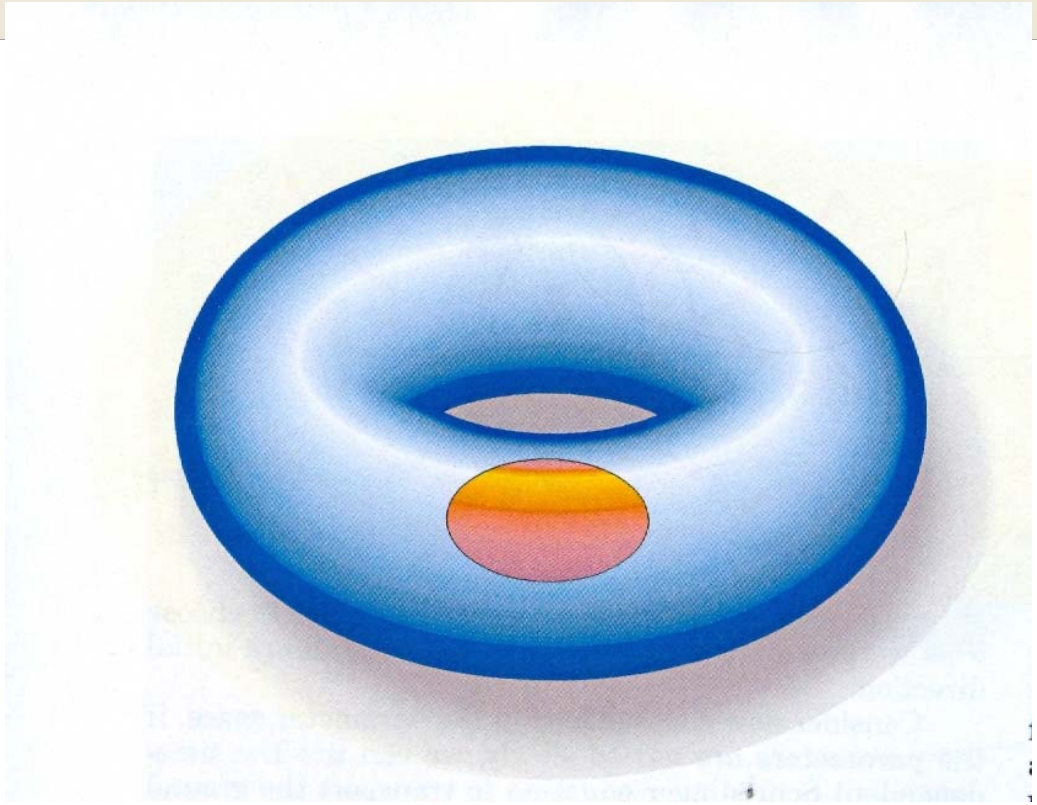




TOPOLOGICAL INVARIANTS

GAUSS

BONNET

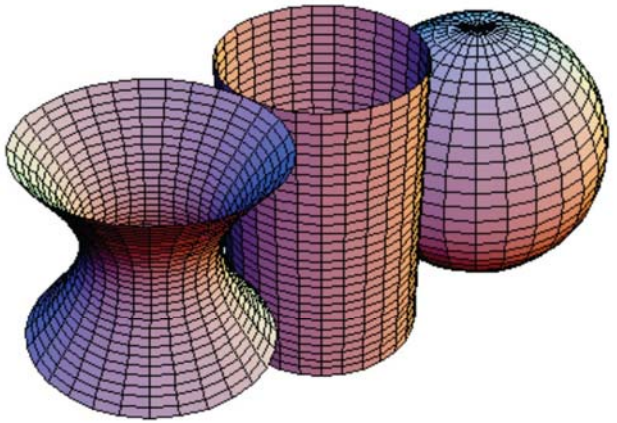


Gauss-Bonnet Theorem

$$\int_S k \, dA = 2\,(1 - g) \quad \text{Mod } 2\pi$$

Surface: $Z(x,y) = \frac{1}{2}\begin{bmatrix} x & y \end{bmatrix} \begin{pmatrix} \frac{\partial^2 Z}{\partial x^2} & \frac{\partial^2 Z}{\partial x \partial y} \\ \frac{\partial^2 Z}{\partial y \partial x} & \frac{\partial^2 Z}{\partial y^2} \end{pmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

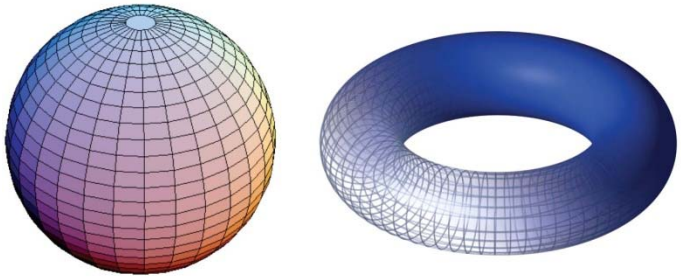
$= \frac{1}{2}\begin{bmatrix} X & Y \end{bmatrix} \begin{pmatrix} \frac{1}{r_1} & 0 \\ 0 & \frac{1}{r_2} \end{pmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}$



from left to right, equators have negative, 0, positive Gaussian curvature

Gaussian Curvature : $\mathcal{K} = \frac{1}{r_1 r_2}$

Closed Surfaces :

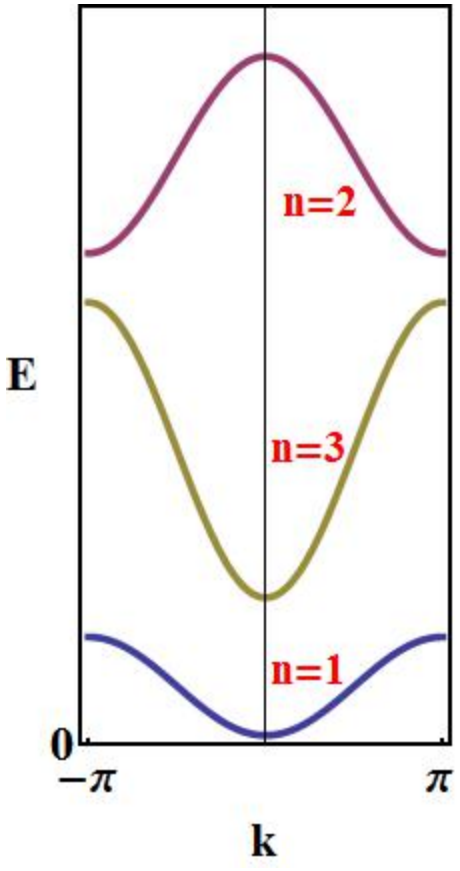


Gauss-Bonnet Theorem

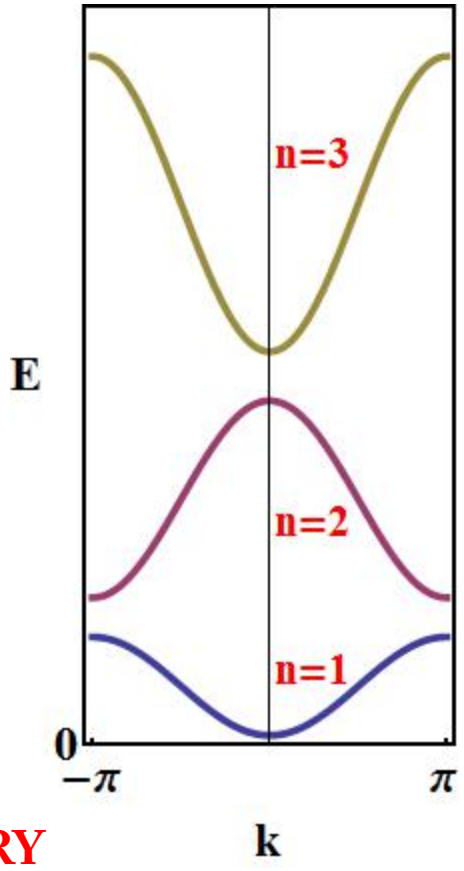
$$\int \mathcal{K}\, dA = 2\pi\ (2 - 2g)$$

where the *genus* $g = 0$ for a sphere, 1 for a torus, n for n-holed torus

TOPOLOGICAL INSULATORS



BOUNDARY

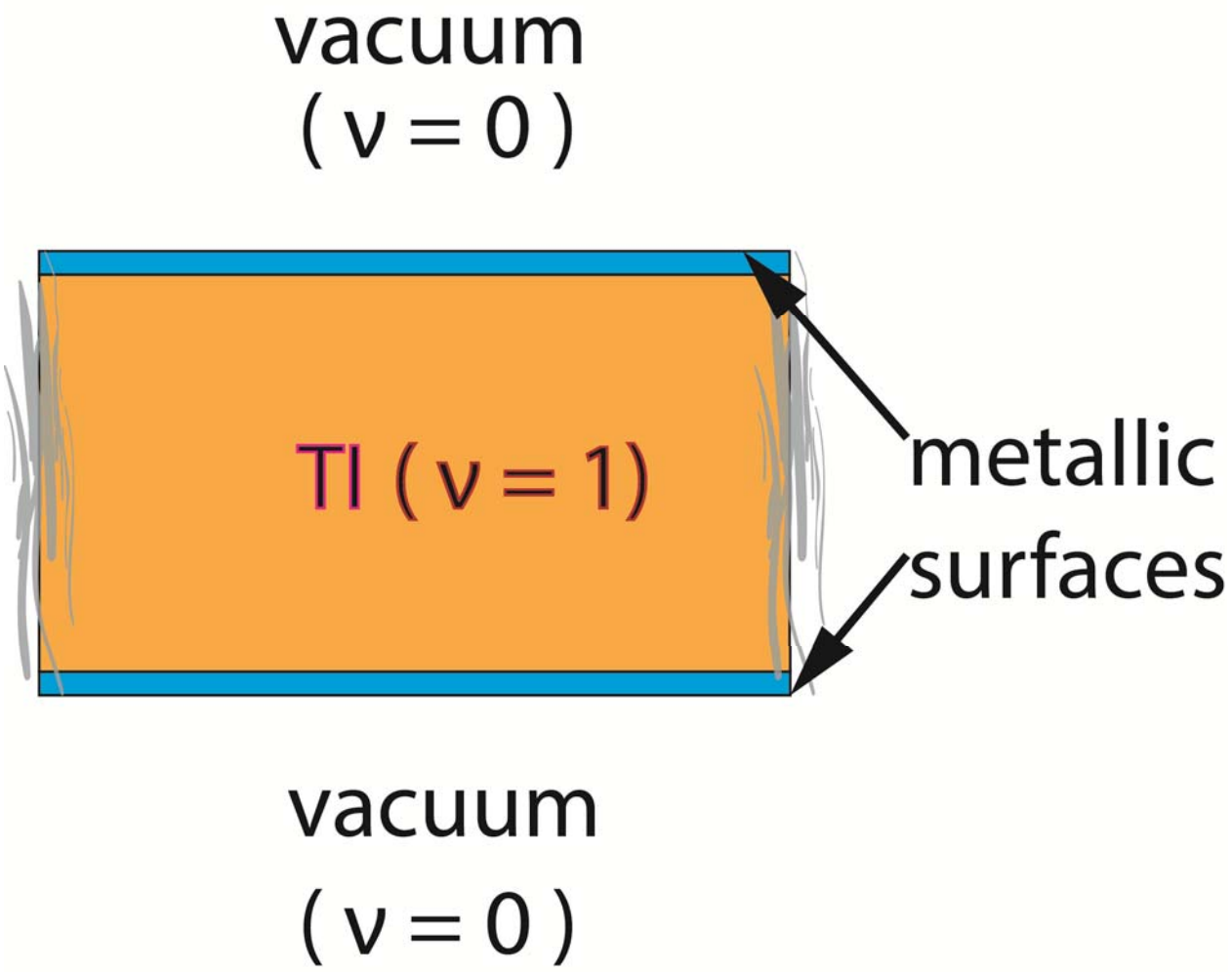


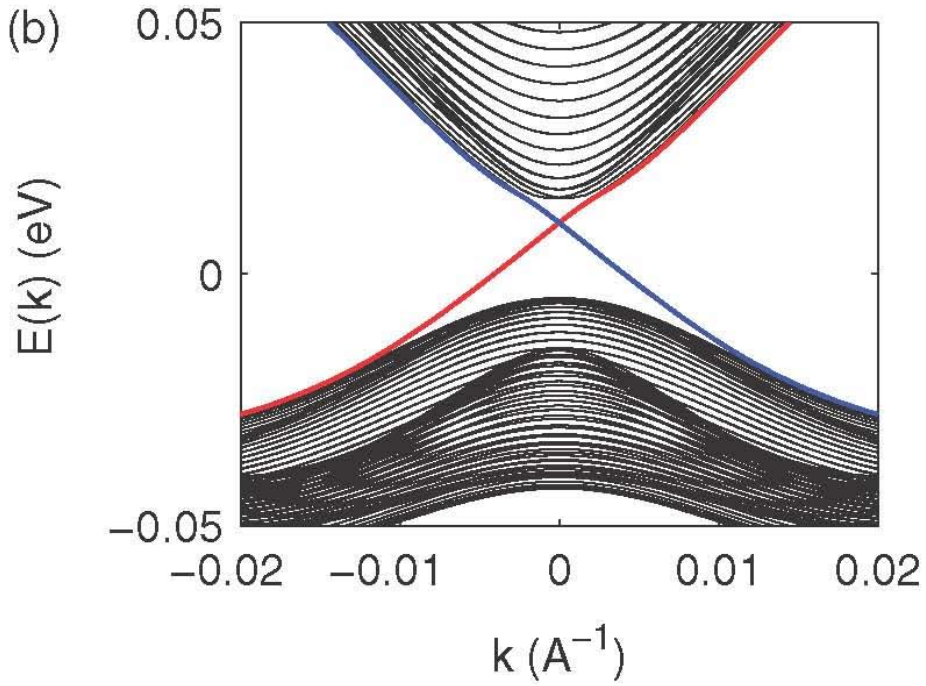
TOPOLOGICAL INSULATOR

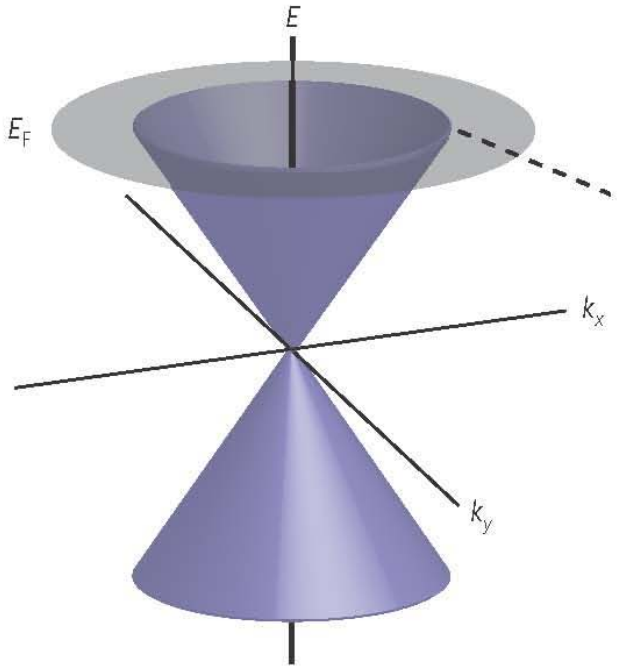
v = 1

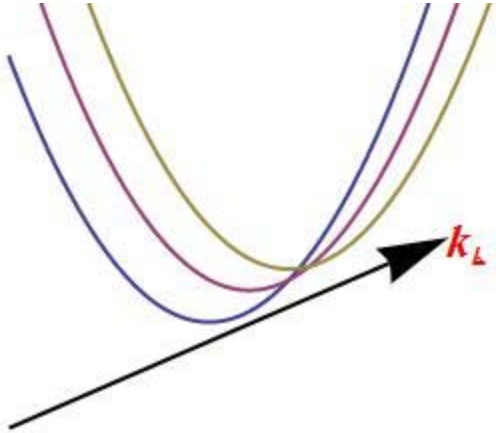
ORDINARY INSULATOR

v = 0


vacuum
(ν = 0)
TI (ν = 1)
metallic
surfaces
vacuum
(ν = 0)

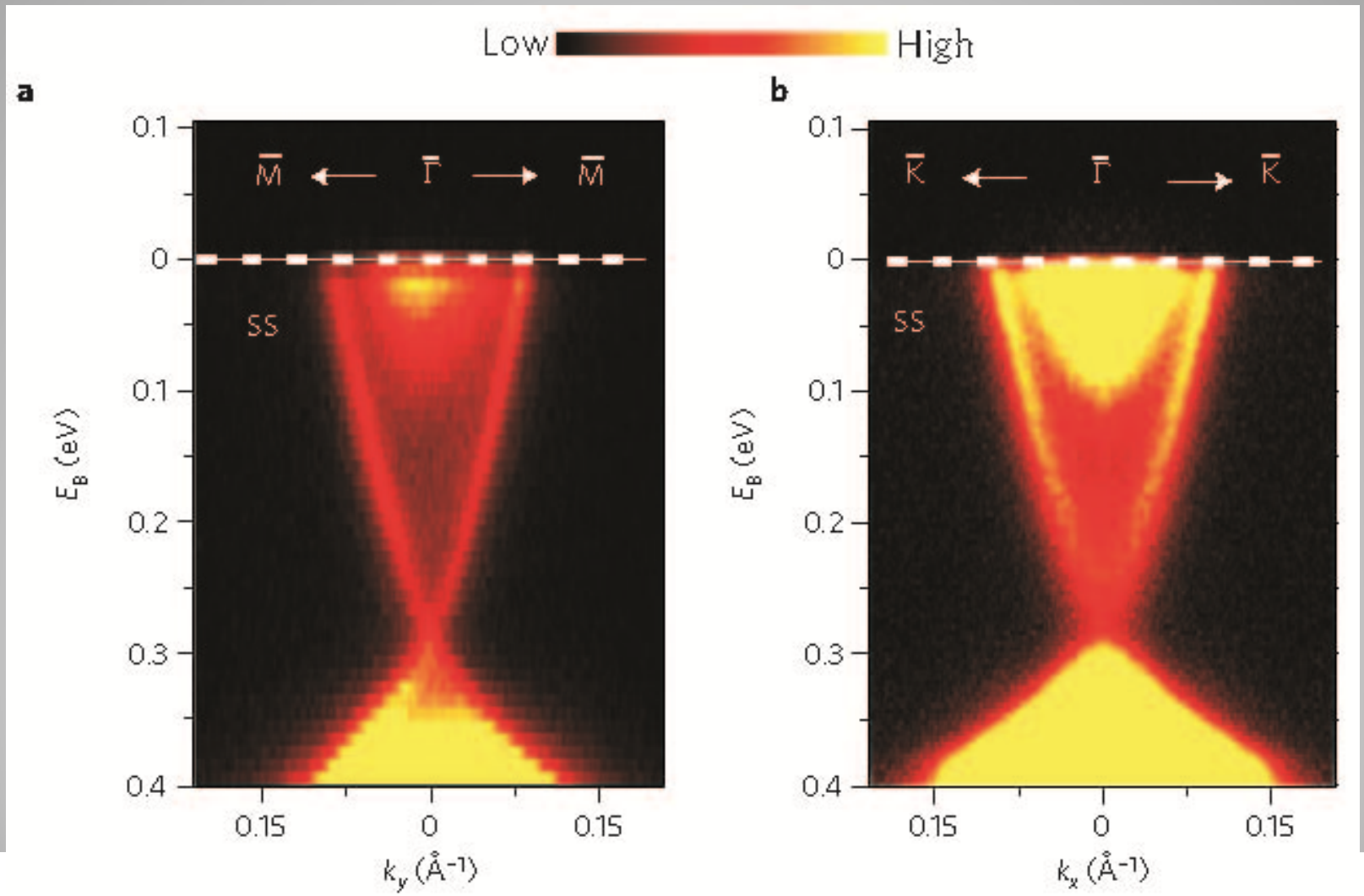






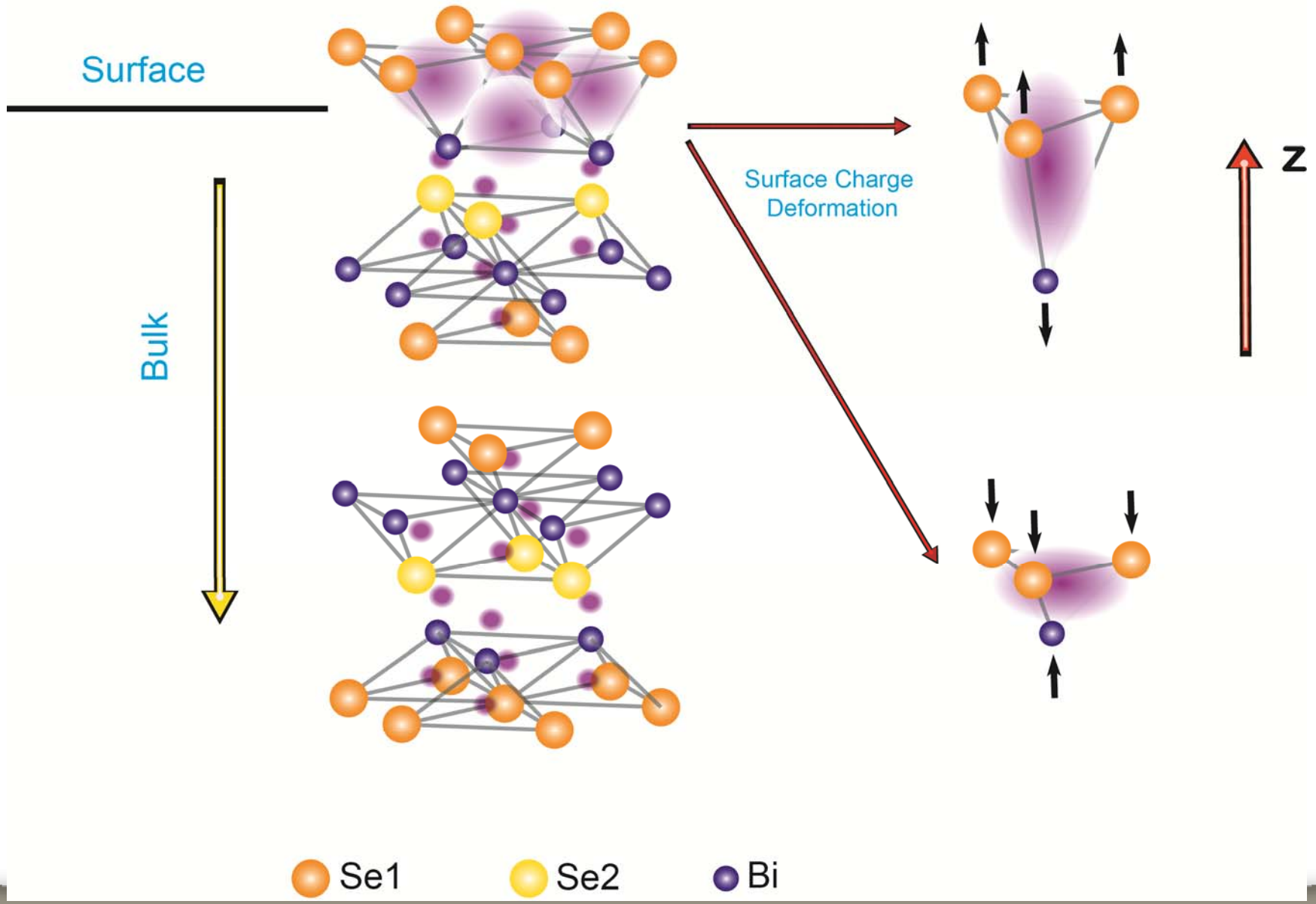
DIRAC CONE

Dirac Cone (Bi_2Se_3)



ELECTRON-ION INTERACTION

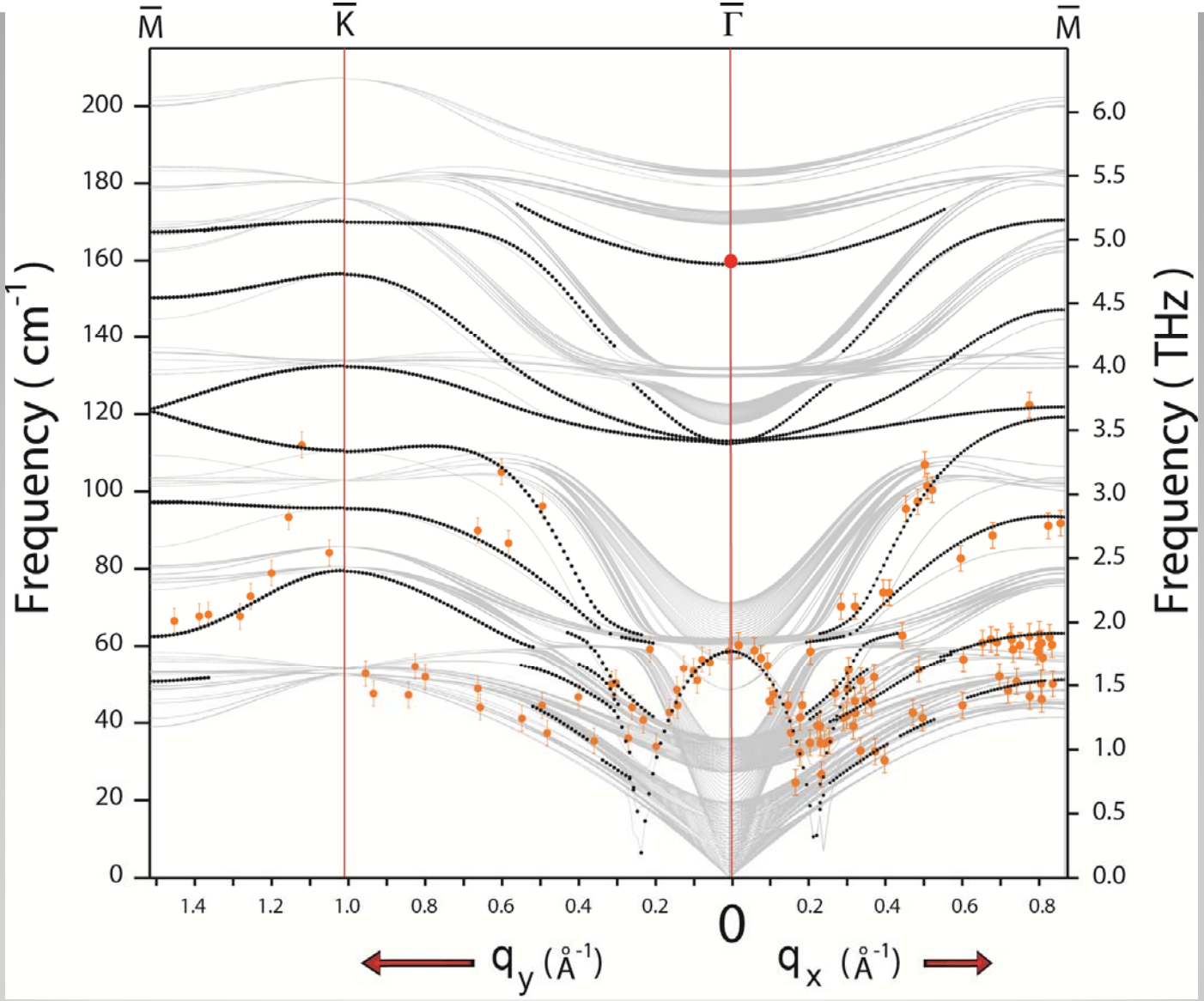
Lattice Dynamics: Pseudo-Charge Model



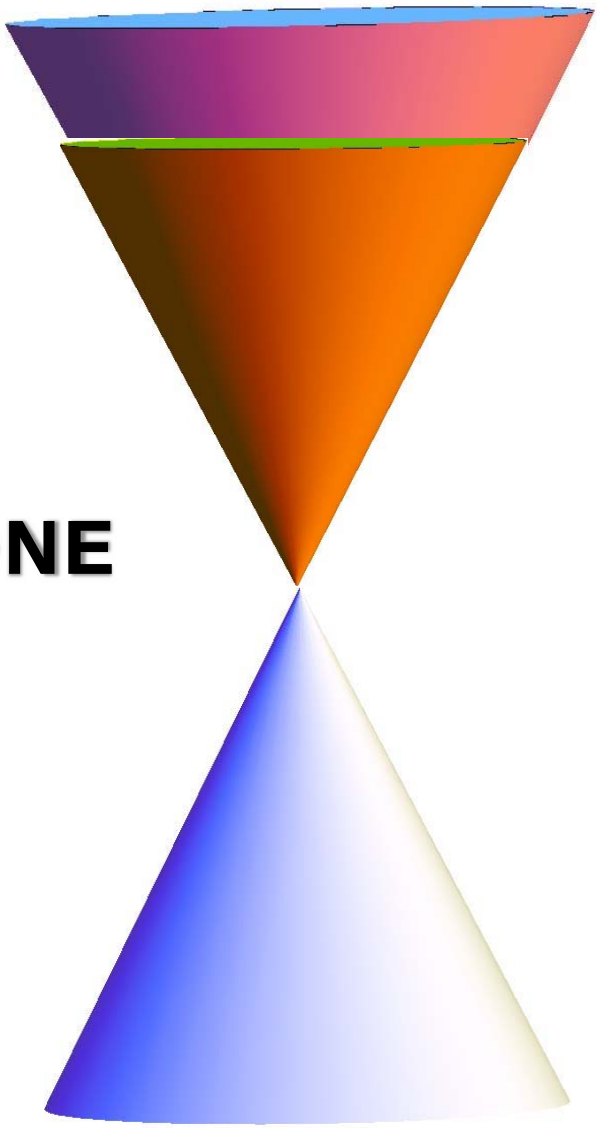
SURFACE PHONON DISPERSIONS

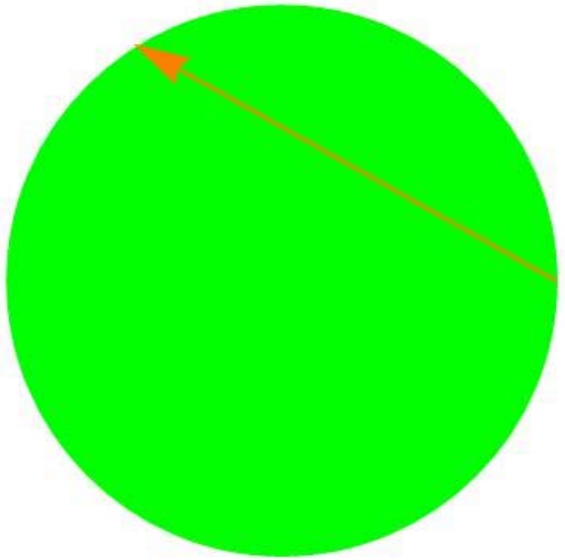
IN THE

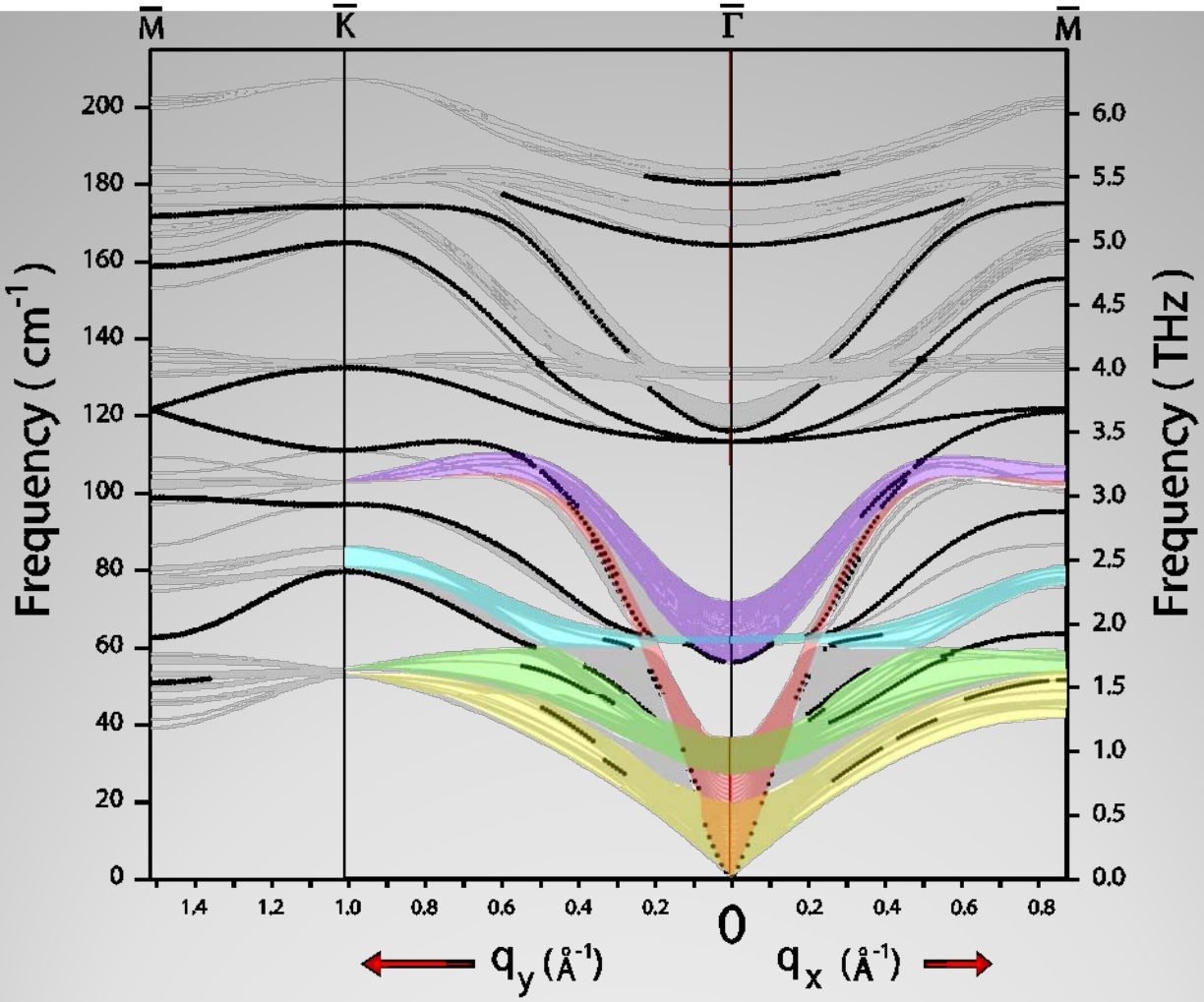
PRESENCE OF DIRAC FERMIONS


M̄
K̄
Γ̄
M̄
Frequency (cm-1)
Frequency (THz)
200
180
160
140
120
100
80
60
40
20
0
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
qy (Å-1)
qx (Å-1)

DIRAC CONE

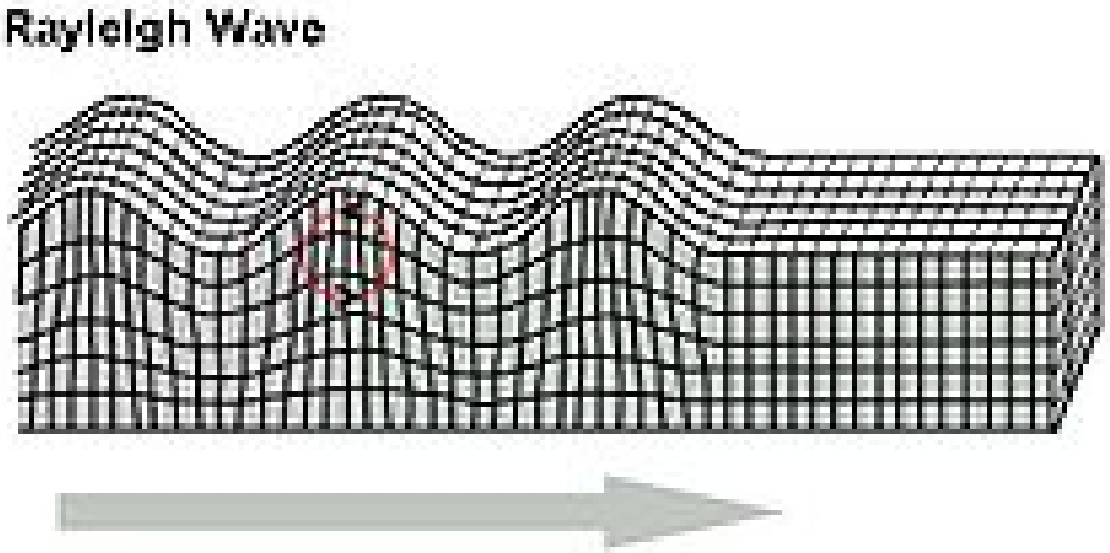




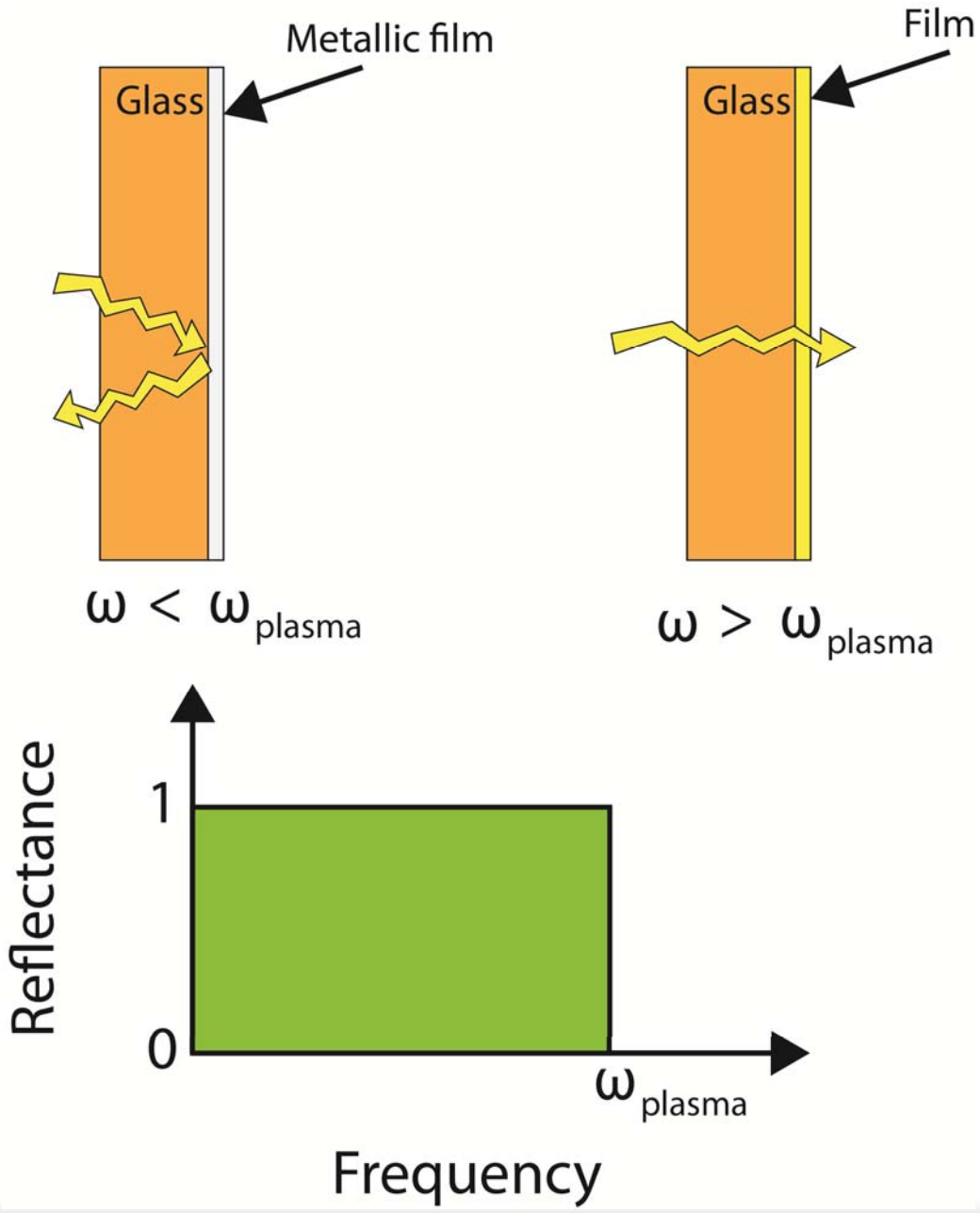


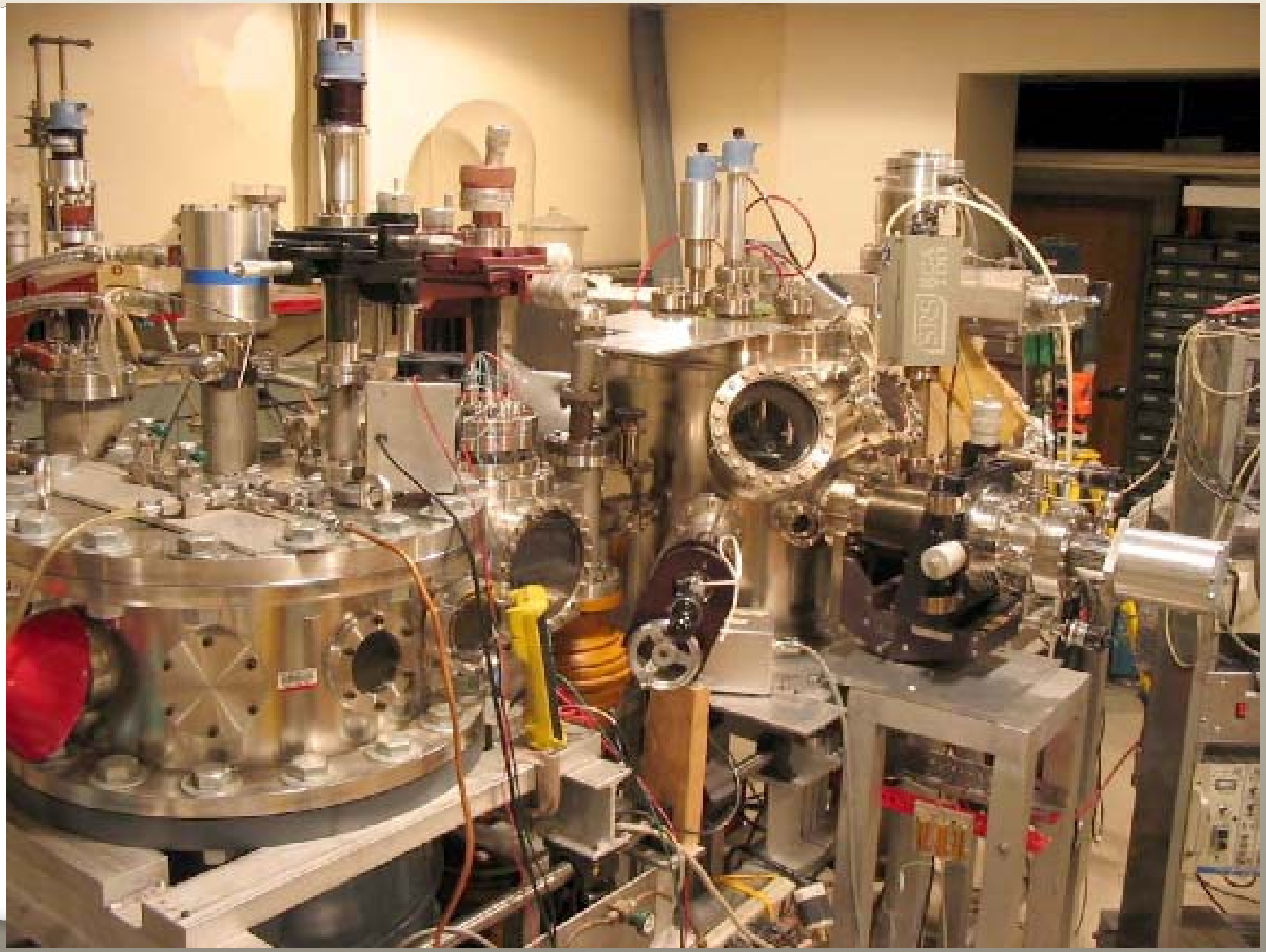
Rayleigh branch for homogeneous surface

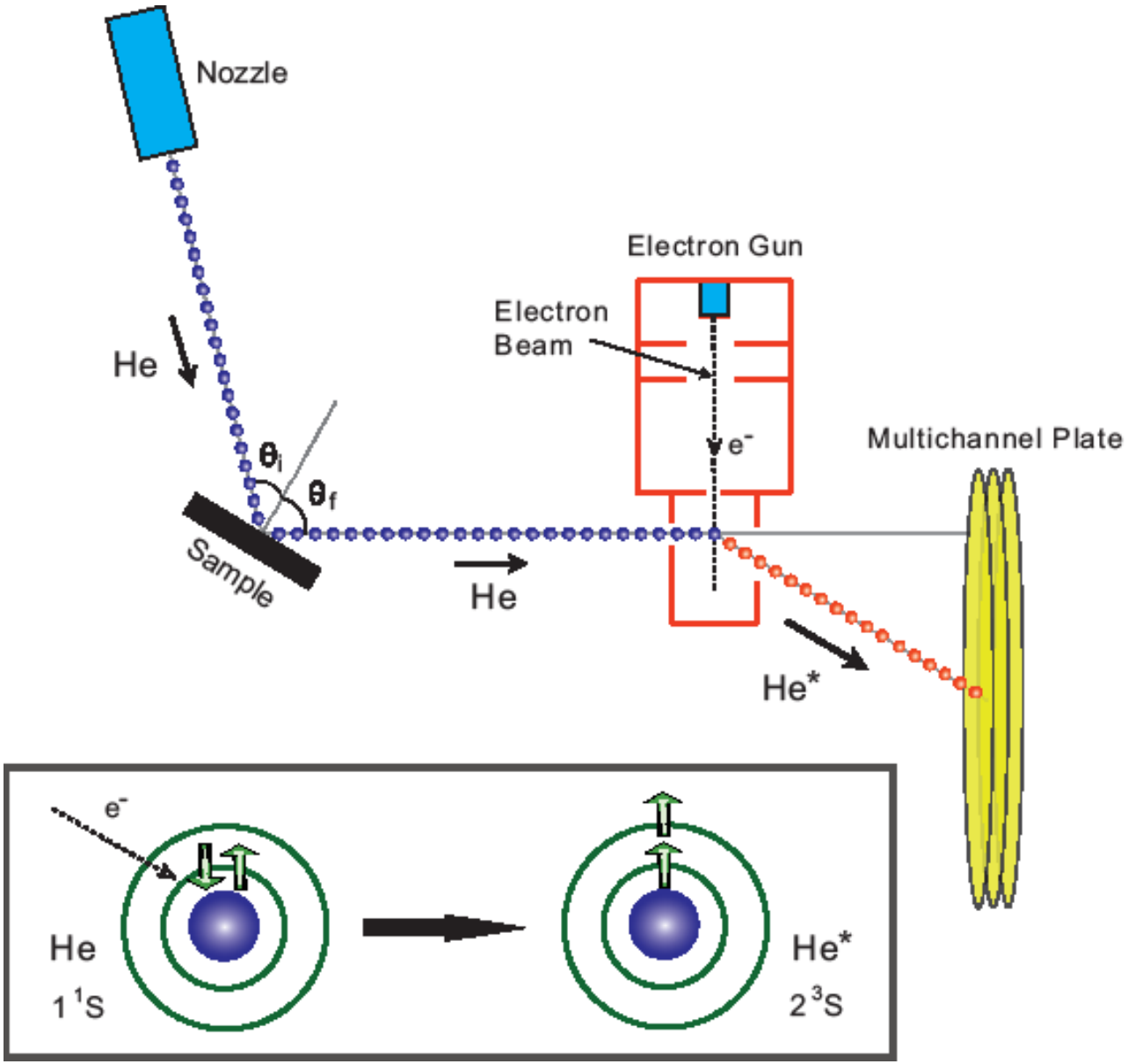
SURFACE RAYLEIGH WAVES

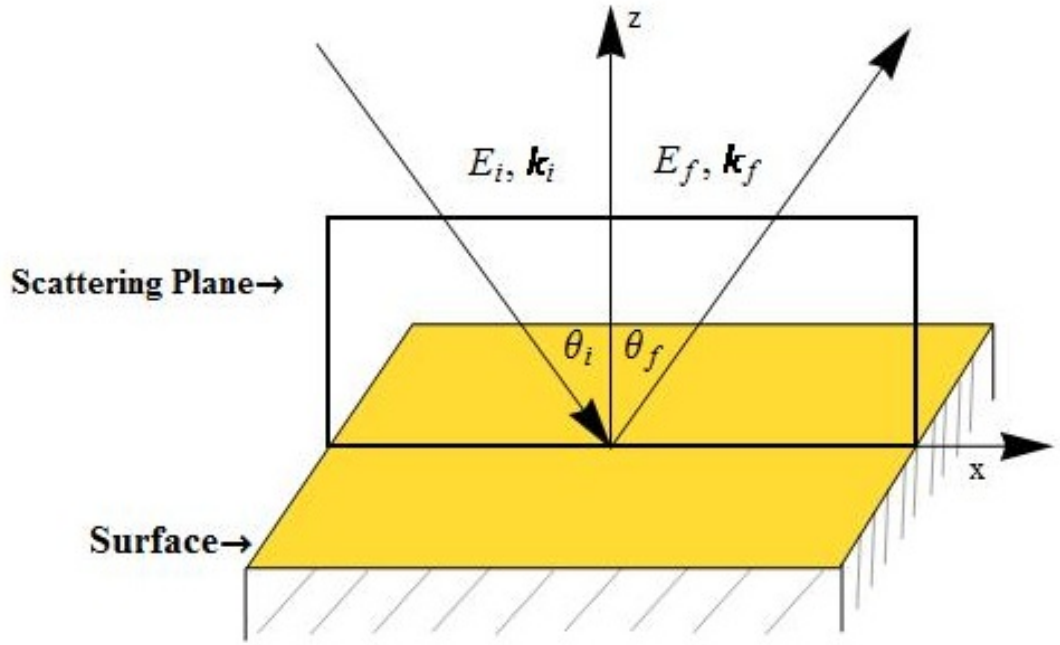


Seismic Waves


Metallic film
Glass
Film
Glass
ω < ω plasma
ω > ω plasma
Reflectance
1
0
ω plasma
Frequency




Nozzle
He
θi
θf
Sample
He
Electron Gun
Electron
Beam
e-
Multichannel Plate
He*
e-
He
1 1S
He*
2 3S



$$E_i - E_f = \hbar\omega \qquad P_f^{\parallel} - P_i^{\parallel} = \hbar k$$