

Errata to: A Guide to First-Passage Processes

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Chapter 1:

1. Page 2: 2nd paragraph should read “To make these questions precise and then answer them, note that the price performs a symmetric random walk in n , where $n = N_- - N_+$ is the difference in the number of “down” and “up” days, N_- and N_+ , respectively.
Thanks to Ronny Straube for this correction.

2. Page 5: 2nd sentence of Sec. 1.2 should read: “An important starting fact is that the survival probability $S(t)$ in an absorbing domain is closely related to the time integral of the first-passage probability up to time t over the spatial extent of the boundary (see Eq. (1.5.7))”.

3. Page 8: The integral in Eq. (1.3.5) is over the range $-\pi \leq k \leq \pi$; it’s not a contour integral.

4. Page 9: 3rd line after Eq. (1.3.6): delete the word “gives”. Eq. (1.3.8) should read

$$P(x, N) \rightarrow \frac{1}{\sqrt{8\pi Npq}} e^{-[x-N(p-q)]^2/8Npq}$$

Thanks to David Waxman, Tibor Antal, and Ehud Yariv for these corrections.

5. Page 9: The 2nd line of the un-numbered formula after Eq. (1.3.8) should read:

$$\sim e^{ik\langle x \rangle - \frac{1}{2}k^2(\langle x^2 \rangle - \langle x \rangle^2)}, \quad k \rightarrow 0,$$

while Eq. (1.3.9) should read:

$$P(x, N) \rightarrow \frac{1}{\sqrt{2\pi N(\langle x^2 \rangle - \langle x \rangle^2)}} e^{-(x-N\langle x \rangle)^2/[2N(\langle x^2 \rangle - \langle x \rangle^2)]}.$$

Thanks to Paul Krapivsky and Ehud Yariv for these corrections.

6. Page 10: In Eq. (1.3.11), the denominator inside the square brackets should be $2zq$ for $x > 0$ and $2zp$ for $x < 0$. Thanks to David Waxman for this correction.
7. Page 11: In Eq. (1.3.13), the left-hand side should read $\frac{\partial P(n,t)}{\partial t}$.
8. Page 11: In Eq. (1.3.14), the factors $-ik$ and ik should be interchanged. The fifth line after this equation should read $w(k) = w_0(\cos k - 1) + 2i \delta w \sin k$. Thanks to Ehud Yariv.
9. Pages 12 & 13: Three lines after Eq. (1.3.19), the inequality should read $n \geq 0$. Two lines after Eq. (1.3.22), there should be a prefactor $\frac{1}{2\pi i}$ before the integral. In the next line, the variable transformation should be $u = \sqrt{-s}$. Thanks to Tibor Antal for these corrections.
10. Page 15: The first line of subsection 1.3.3.2 should read: “Here we solve the convection-diffusion equation (1.3.23) by ...”
11. Page 17: In Eq. (1.3.33), in the line before, and in the figure, the factor i should be replaced by $-i$. Thanks to Ehud Yariv.
12. Page 20: In the 3rd line from the end of the page, (1.4.9) should be (1.4.10).

13. Page 21: In the 2nd line of (1.4.12), the factor dt is missing. Thanks to Ehud Yariv.
14. Page 22: In Eq. (1.5.6), the factor $(1 - R)$ should read $(1 - \mathcal{R})$.
15. Page 24: Seventh line from the end, replace “when” by “where”. Thanks to Ehud Yariv.
16. Page 31: In Eq. (1.6.22), the argument on the left-hand side should be \vec{r} , not \vec{r}' .

Chapter 2:

1. Page 46: The statement “complete parallelism” in the 4th line is a bit misleading. While the splitting probabilities and the exit times for the discrete random walk and continuum diffusion agree, this correspondence does not extend to higher moments. Thanks to Tibor Antal.
2. Page 47: The factor in the 2nd of Eqs. (2.2.16) should be $(1 - u_0^2)$ not $(1 - u_0)^2$.
3. Page 48: Wald’s identity applies to a random-walk process with continuous step lengths that occur at discrete times. Thus the quantity $\langle t(x_0) \rangle$ in Eq. (2.2.19) should be interpreted as the *number* of steps until absorption, not the time to absorption. With this interpretation for $\langle t(x_0) \rangle$, the last of Eqs. (2.2.19) is dimensionally correct. Thanks to Ming Ma.
4. Page 49: In the 2nd line of Eq. (2.2.23), x_0^2 should appear inside the square root. This same correction applies on the last line of this page. In the 2nd-to-last line: $j_-(s; x_0)$ instead of $j(s; x_0)_-$. Thanks to Shai Carmi and Ehud Yariv.
5. Page 50: On the 5th and 6th line, the factor $L - x_0$ should be $L - x_0/2$. Thanks to Ehud Yariv.
6. Page 51: Five lines before (2.2.24), the factor $\delta(t)$ should be $-\delta(t)$. In the next line, as well as one line before (2.2.24), $j(L, s) = 1$ should read $j(L, s) = -1$. Eq. (2.2.24) is correct as written. In the line just below (2.2.24), the condition should read “ x_0 set to L ”. In this case $x_> = L$ and $x_< = x$ in Eq. (2.2.20) so that it coincides with (2.2.24). Thanks to Ehud Yariv.
7. Page 52: The prefactor in the last line of (2.2.26) is upside down and the sign is wrong. This prefactor should read $(-1)^n \frac{(2n+1)\pi D}{L^2}$. Thanks to Maciej Dobrzynski and Gleb Oshanin.
8. Pages 53 & 54: The last line on page 53 should read “and the Laplace transform of the initial condition, $j(x = 0, s) = 1$, fixes the constant to be”. The line immediately after the formula at the bottom of page 53 should be followed by: “where we have used the simplification $v - D\alpha_{\pm} = D\alpha_{\mp}$ ”. The first line of page 54 should read: “The Laplace transform of the outlet flux, or equivalently, the Laplace transform of the first-passage probability, is”. There no D in the denominator of Eq. (2.2.29). The statement immediately after (2.2.29) should then be removed. The 5th line of text should read: “where $Pe = \frac{vL}{2D}$ is again the Péclet number and $P_s \equiv \sqrt{v^2 + 4Ds} \frac{L}{2D}$ ”. Thanks to Shai Carmi for some of these corrections.
9. In Eq. (2.2.29), the last term in the denominator should read $\alpha_- e^{-\alpha_+ L}$ and the overall factor of D in the denominator should be removed. The line immediately after (2.2.29) is used to obtain the last equation on page 53, rather than (2.2.29). In the next line, remove the word “dimensionless”. Thanks to Eli Ben-Naim and Ehud Yariv.
10. Page 55: The displayed equation and the line following should read:

$$\tanh \sqrt{Pe^2 + sL^2/D} = \sqrt{1 + sL^2/(D Pe^2)}$$

which, in the limit $Pe \rightarrow \infty$, gives the criterion $s \approx -(4DPe^2/L^2) e^{-2|Pe|} \equiv 1/\tau$.

11. Page 56 after (2:3.5) should read $\langle t(x_0)^2 \rangle = 2 \int_0^L C_1(x) dx$, *i.e.*, (factor 2 is missing).
12. Page 57: In the 2nd line, the signs of the v and D terms are *not* opposite to the sense of the corresponding terms in the convection-diffusion equation. Also, 5 lines from the bottom, the length scale is D/v not D/v^2 .
13. Page 58, first equation, 2nd line, the factor D equals $\delta x^2/2\delta t$, *i.e.*, a factor 2 is missing.
14. Page 59: The solid curve in Fig. 2.5 should be labeled $2.5 t_{\max}/\tau$ and the last caption phrase should read “(multiplied by 2.5 for visibility)”. The sentence just before (2.3.13) should read: “The corresponding first-passage time is (with $z \equiv x_{\max}/L$)”, while the factor Pe at the end of the first line of (2.3.13) should be smaller. Thanks to Shai Carmi for this and the previous 4 corrections.
15. Page 63: Just above (2.3.23), the reference is to (1.6.27) not (1.6.29).
16. Page 64: Two lines above (2.3.24), the boundary conditions should be $g_+(0) = g_+(2Pe) = 0$. Thanks to Shai Carmi.
17. Page 64: Eq. (2.3.25) should read

$$t_-(x) = \frac{x}{v} \left(\frac{1 + e^{-v(x-L)/D}}{1 - e^{-v(x-L)/D}} \right) - \frac{2L/v}{1 - e^{-v(x-L)/D}} \left(\frac{1 - e^{-vx/D}}{1 - e^{-vL/D}} \right).$$

That is, each exponent in this formula has a minus sign compared to (2.3.25).

18. Page 71: There should be a factor w multiplying $P_1(t)$ in the last of the un-numbered equations at the bottom of the page.
19. Page 74: The 2nd-to-last of Eqs. (2.4.10) should read $P_N = \frac{a}{1-z} P_{N-1}$; *i.e.*, the letter z should be lower case.
20. Page 75: While (2.4.12) is correct, some text is sloppy. Two lines above (2.4.11), the text should read “This form is valid for $n = 2, 3, 4, \dots, N$, whereas the equations for P_0 and P_1 are distinct.” Parenthetically, to reach the un-numbered displayed equation after (2.4.11) one can use tricks like $2a\lambda_{\pm} - 1 = \pm\sqrt{1 - 4a^2}$ as well as $\sqrt{1 - 4a^2} = a(\lambda_+ - \lambda_-)$. Thanks to Jakub Otwinowski for clarifying these points.
21. Page 75: Line after (2.4.12): replace “completely” by “asymptotically”.
22. Page 76: The line after (2.4.14) should read: “Continuing this procedure to site $N - 2$ gives” ($N - 2$ instead of $N - 1$), and in the next equation replace P_{N-1} on the left-hand side with P_{N-2} and P_N in the 2nd term of the right-hand side with P_{N-1} . The next line of text should be: “Then the equation for P_{N-1} gives”, and the following equations should read: $P_{N-1} = aP_{N-2} = \dots + f_{N-1}P_{N-1}$ or $P_{N-1} = \dots$. Thanks to Shai Carmi for these corrections.

Chapter 3:

1. Page 81: For $t \rightarrow \infty$, the exponential at the end of Eq. (3.2.2) should be replaced by 1. Thanks to Ehud Yariv.
2. Page 83: In Eq. (3.2.4) the derivative should be with respect to x not t and the final result applies for all times. Five lines after (3.2.4), the correct statement is $F(0, t) \sim x_0/\sqrt{4\pi Dt^3}$. Thanks to Carl Gold and Ehud Yariv.

3. Page 84: Two lines after (3.2.5) replace $F(t)$ by $F(0, t)$. In Eq. (3.2.6) there is no factor of t in the square root. Thanks to Robin Groenevelt and Ehud Yariv.

4. Page 86, Eq: (3.2.11). Oy! Is this formula messed up! (3.2.11) is

$$\langle x \rangle = \frac{1}{S(t)} \frac{1}{\sqrt{4\pi Dt}} \int_0^\infty x \left[e^{-(x-x_0)^2/4Dt} - e^{-(x+x_0)^2/4Dt} \right] dx.$$

In the first integral, define $y = (x - x_0)/\sqrt{4Dt}$ and in the second define $y = (x + x_0)/\sqrt{4Dt}$. Then (3.2.11) becomes

$$\langle x \rangle = \frac{1}{S(t)\sqrt{\pi}} \left[\sqrt{4Dt} \left(\int_{-z_0}^\infty - \int_{z_0}^\infty \right) y e^{-y^2} dy + x_0 \left(\int_{-z_0}^\infty + \int_{z_0}^\infty \right) e^{-y^2} dy \right],$$

where $z_0 = x_0/\sqrt{4Dt}$. The first two integrals give zero, by symmetry, while the last two integrals give

$$\langle x \rangle = \frac{2x_0}{S(t)\sqrt{\pi}} \left[\int_0^{z_0} e^{-y^2} dy + \int_{z_0}^\infty e^{-y^2} dy \right] = \frac{2x_0}{S(t)\sqrt{\pi}} \left[\int_0^\infty e^{-y^2} dy \right] = \frac{x_0}{S(t)} \sim \sqrt{\pi Dt}.$$

Thanks to Shai Carmi for the original comment. Later improvements by me.

5. Page 87: In Eq. (3.2.12) the prefactor in the 2nd term should be $e^{-vx_0/D}$. Similarly, in the first and also the 3rd line of text below this equation the factor should read $e^{-vx_0/D}$.

6. Page 88: On line 9, the flux should read $-(vc - D \frac{\partial c}{\partial x})$. In the line after Eq. (3.2.14), $u^2 = x^2/4Dt'$. Thanks to Shai Carmi and Ehud Yariv.

7. Replace x by x_0 and t by t' in the line after Eq. (3.2.14) and in the first line of (3.2.15). Also, the fontsize for the first factor of Pe in the 3rd line of this formula should be larger.

8. Eq. (3.2.15) can be evaluated more simply (using Mathematica):

$$\begin{aligned} S(t) &= 1 - \frac{2}{\sqrt{\pi}} e^{Pe} \int_{x_0/\sqrt{4Dt}}^\infty e^{-u^2 - Pe^2/4u^2} du \\ &= 1 - \frac{1}{2} e^{-Pe - |Pe|} \left[2 - \operatorname{erfc}\left(\frac{|Pe|}{2z} - z\right) + e^{2Pe} \operatorname{erfc}\left(\frac{|Pe|}{2z} + z\right) \right] \end{aligned}$$

where $z \equiv x_0/\sqrt{4Dt}$. For $t \rightarrow \infty$ ($z \rightarrow 0$), the above result for $Pe > 0$ reduces to the first line of (3.2.16). For $Pe < 0$, the asymptotic expansion of the error function leads to

$$\begin{aligned} S(t) &= \frac{1}{2} \operatorname{erfc}(|Pe|/2z - z) - \frac{1}{2} e^{2Pe} \operatorname{erfc}(|Pe|/2z + z) \\ &\simeq \frac{1}{\sqrt{4\pi}} \left[\frac{e^{-Pe^2/4z^2 + |Pe| - z^2}}{(|Pe|/2z - z)} - e^{2|Pe|} \frac{e^{-Pe^2/4z^2 - |Pe| - z^2}}{(|Pe|/2z + z)} \right] \\ &= \frac{1}{\sqrt{4\pi}} e^{-(|Pe|/2z - z)^2} \left[\frac{1}{(|Pe|/2z - z)} - \frac{1}{(|Pe|/2z + z)} \right] \\ &\simeq \frac{1}{\sqrt{4\pi}} \frac{8z^3}{Pe^2} e^{-Pe^2/4z^2} e^{|Pe|} = \sqrt{\frac{4}{\pi}} \frac{x_0 \sqrt{Dt}}{(vt)^2} e^{|v|x_0/2D} e^{-v^2 t/4D} \end{aligned}$$

which replaces the 2nd line of (3.2.16). Thanks to David Mukamel, Ehud Yariv, and Benjamin De Bruyne.

9. Page 93: The prefactor in Eq. (3.3.1) should be $\frac{1}{\sqrt{4Ds}}$. In the line below (3.3.1), $c_>$ and $c_<$ should be interchanged. Page 94: In the first line of Eq: (3.3.3), the last factor should be $e^{-\alpha+x_0}$. Thanks to Shai Carmi for latter two corrections.
10. Page 99: Last two lines of Eq. (3.4.4): the denominator factor should be $2(t-1)$ not $2t-1$. The last factor in Eq. (3.4.5) should be $\sqrt{\frac{2}{\pi t}} 2^t$. Thanks to Shai Carmi for the latter.
11. Page 102: The first line of Eq. (3.4.10) should read Thanks to Shai Carmi.

$$\frac{P(x, z)}{P(0, z)} \left[1 - \frac{1}{P(0, z)} \right]^{m-1}$$

12. Page 103: The 2nd argument of F in Eq. (3.4.12) should be j , not n . In the line above Eq. (3.4.14), the references should be to Eq. (1.2.3), not (1.2.1). In Eq. (3.4.14), the exponent should be $m-1$, not m . Finally, the equivalence between Eqs. (3.4.14) and (1.3.11) mentioned at the bottom of the page holds *only* in the limit $z \rightarrow 1$ (and up to an overall factor of $\sqrt{2}$). Thanks to Shai Carmi for these corrections.
13. Page 103: Eq. (3.4.16) should read

$$G^{(m)}(0, n) \sim \sqrt{\frac{2}{\pi n}} e^{-m^2/2n}$$

14. Pages 104–5: In the caption to Fig. 3.9, the coordinate x refers to the position of a random walk. However on page 105, x in Eq. (3.4.18) refers to the scaled position $x = k/n$. This relation is belatedly stated just before Eq. (3.4.19). Finally, in going from (3.4.17) to (3.4.18), we should write $Q_{n,k} dk = Q(x)dx$, with $x = k/n$. With this properly scaled definition, there is no factor of $1/n$ on the right-hand side of Eq. (3.4.18). Thanks to Irving Herman.
15. Page 105: Three lines above Eq. (3.4.17), the reference should be to Eq. (3.4.7). The 2nd line of Eq. (3.4.17) should read

$$\frac{2}{\pi \sqrt{k(n-k)}}.$$

Also, the limits of integration in Eq. (3.4.19) should be from k to n . Thanks to Shai Carmi.

16. Page 108: Add the word “with” in the 9th line. It should read “... returned to site 1 with probability $r \dots$ ”. Thanks to Ronny Straube.
17. Page 114: In the 2nd to last line, the slope is c_0/\sqrt{Dt} Thanks to Shai Carmi.

Chapter 4:

1. Page 123: Sec. 4.3.2, 3rd line: “Flyvbjerg”, not “Flyvbjery”.
2. Page 130: The last term in the equation at the top of the page should read $\frac{1}{2}(r_{\text{in}} + r_{\text{out}}) \frac{\partial^2 P}{\partial n^2}$. Thanks to Paul Krapivsky.

3. Page 142: Eqs. (4.5.5) should read: Thanks to Nan Shi for these corrections.

$$\langle t \rangle = \frac{D}{v_1 v_2} \left\{ (1 - e^{-v_1 x_1 / D}) \left[1 - e^{v_2(x_1-1)/D} \right] + \frac{v_2 x_1}{D} + \frac{v_1(1-x_1)}{D} \right\} \\ + \frac{D}{v_1^2 v_2^2} \left\{ v_2^2 \left[e^{-v_1 x_1 / D} - 1 \right] + v_1^2 \left[e^{v_2(x_1-1)/D} - 1 \right] \right\},$$

while the correct form of (4.5.6) is:

$$\langle t \rangle \rightarrow \begin{cases} \frac{x_1}{v_1} + \frac{1-x_1}{v_2}, & v_1, v_2 \rightarrow +\infty, \\ \frac{D}{|v_1 v_2|} e^{[|v_1| x_1 + |v_2| (1-x_1)]/D} & v_1, v_2 \rightarrow -\infty. \end{cases}$$

4. Page 151–152: The sign of the 3rd term on the right-hand side of Eq. (4.6.1) should be plus, not minus. The prefactor on the right side of Eq. (4.6.5) should be 1 not 4, and the prefactor on the right side of Eq. (4.6.6) should be $\frac{1}{2}$ not 2. Remove the word “a” four lines after Eq. (4.6.6). Thanks to Alex Petersen for the first 3 corrections.
5. Page 160–161: The left-hand side of Eq. (4.7.15) should be $\ln S(x)$. In Eq. (4.7.17), the correction term in the integrand should include a factor of $\frac{1}{2}$.
6. Page 166: The double subscript y_{y_1} should be y_1 . Thanks to Robin Groenevelt.

Chapter 5:

1. Page 171: Eq. (5.2.4) is appropriate for a discrete-time random walk, but the process defined in Eq. (5.2.1) is in continuous time. Thus (5.2.4) should read

$$P_n(t) = \int_0^t F_n(t') P_0(t-t') dt' + \delta_{n,0} e^{-t}$$

The exponential factor in the 2nd term gives the probability that the walk has not moved before time t , in which the hopping rate along each bond is $1/q$. After performing the Laplace transform, the correct form of (5.2.5) is

$$F_n(s) = \frac{P_n(s) - \delta_{n,0}/(s+1)}{P_0(s)}$$

Similarly, the correct form of (5.2.6) is

$$F_0(s) = 1 - \frac{1}{(s+1)P_0(s)}$$

The fundamental $s \rightarrow 0$ results in the next two formulae remain unchanged.

2. Page 196: The third to last line of section 5.5.1.1 should have the factor ℓ_3^{2k-1} .
3. Page 202: Seventh line in section 5.5.3.1, a factor should read $1/(3+2\epsilon)$, not $1(3+2\epsilon)$.

Chapter 6:

1. Page 209: The result in the 2nd line of Eq. (6.2.2) actually gives $\mathcal{E}_-(r)$. The correct result is:

$$\mathcal{E}_+(r) = \frac{\ln(r/R_-)}{\ln(R_+/R_-)}.$$

Thanks to Ronny Straube for this correction.

2. Page 211: In Eq. (6.2.5b), the last line is for $d > 2$; also the last factor should be R_- , not R_+ . Thanks to Keith Cheveralls for the first correction and Mika Pruikkonen for the second.
3. Page 215: In the line after Eq. (6.3.3b), the factor should read $(a/r_0)^{d-2}$.
4. Page 219: Just before the first displayed (and un-numbered equation), the reference to the equations in chapter 2 should be Eqs. (2.2.7) and (2.2.8).
5. Page 219: The formulae for the derivatives of the Bessel functions before Eq. (6.4.3) are wrong. They should read: $I'_\nu = -\frac{\nu}{x}I_\nu + I_{\nu-1}$ and $K'_\nu = -\frac{\nu}{x}K_\nu - K_{\nu-1}$.
6. Page 220: Eq. (6.4.5) has some small errors. The first line should contain a minus sign after the equivalence. In the 3rd line, the leading \pm should be replaced by \mp and the exponent ν should be replaced by $-\nu$. Thanks to Enrique Abad.
7. Page 221-2: The last sentence of page 221 and the first sentence of page 222 should be replaced by: “The factor s^μ in the leading correction term means that the hitting probability at time t varies as $t^{-(1+\mu)} = t^{-d/2}$. For $d > 4$, the time dependence of the hitting probability decays faster than t^{-2} for long times, so that the mean hitting time for the returnees is finite.”
8. Page 226: The paragraph after Eq. (6.5.5c) should be placed immediately after Eq. (6.5.5b).
9. Pages 226–227: In the first formula in subsection 6.5.2.2 and in the first un-numbered formula after Eq. (6.5.6), the prefactor should be $\sqrt{\frac{D}{s}}$ rather than $\frac{D}{s}$. Thanks to Ronny Straube.

Chapter 7:

1. Page 236: First sentence of the 3rd paragraph: the phrase “We thus define $c(r, \theta, t = 0) \dots$ ” is missing an equal sign.
2. Page 240: The exponent in Eq. (7.3.2) should be ν , not ν_0 .
3. Page 243: In the first line of Eq. (7.4.3), the fraction $\frac{1}{2\pi D}$ should be replaced by D . Thanks to Robin Groenevelt for these corrections.

Chapter 8:

1. Page 261: In Eq. (8.2.10) the factor $\ln w$ in the exponential should read $\ln q$ and not $\ln w$.
2. Page 266: The expressions for Θ_{end} and β_{end} should read: Thanks to Alan Bray for these corrections.

$$\Theta_{\text{end}} = \pi - \cos^{-1} \frac{D_3}{\sqrt{(D_1 + D_3)(D_2 + D_3)}},$$

$$\left[2 - \frac{2}{\pi} \cos^{-1} \frac{D_3}{\sqrt{(D_1 + D_3)(D_2 + D_3)}} \right]^{-1}.$$

3. Page 269: In figure 8.6(a) the labels $1 = 3$ and $2 = 3$ should be transposed.
4. Page 284: In Eq. (8.4.24), the time-dependent prefactor should be $t^{-3/2}$, not $t^{-1/2}$. Thanks to Pu Chen for this correction.