Errata: A Kinetic View of Statistical Physics

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Chapter 2:

1. Page 13: The line above Eq. (2.3) should read: "…that $P_N(x) = \Pi_N[r = (x+N)/2)]|dr/dx|$ becomes[2]". Then in Eq. (2.3), there should be a factor of $8Npq$ in the exponent, not $2Npq$, and a factor of 8π in the leading square root, not 2π. (Thanks to James Silva, Tibor Antal, and David Liu.)

2. Page 27. As written, Eq. (2.44) is appropriate for a discrete-time random walk. For a continuous-time random walk with a unit hopping rate between neighboring sites, which is what we actually consider, the factor $\delta(t)$ in Eq. (2.44) should be replaced by e^{-2dt}, the probability that there is no hopping by time t (here d is the spatial dimension). The error in Eq. (2.44) propagates in a straightforward way throughout this section, as listed below: (Thanks to Tibor Antal.)

   (a) The in-line equation just above (2.45) should be $P(\mathbf{r}, s) = F(\mathbf{r}, s)\, P(\mathbf{0}, s) + \delta_{\mathbf{r},\mathbf{0}}/(s+2d)$, and Eq. (2.45) becomes

   $$F(\mathbf{r}, s) = \frac{P(\mathbf{r}, s) - \delta_{\mathbf{r},\mathbf{0}}/(s+2d)}{P(\mathbf{0}, s)} .$$

   (b) For clarity, the un-numbered equation at the top of page 28 should be written as

   $$\mathcal{R} = F(\mathbf{0}, s=0) = \int_0^\infty F(\mathbf{0}, t)\, dt\,;$$

   (c) Two lines above (2.49) should read "probability at the origin, in the limit $s \to 0$, is $P(s) \simeq 1/\sqrt{4s}$.", and Eq. (2.49) itself should be

   $$F(s) \simeq 1 - \sqrt{s}\,.$$

   (d) The last factor in the first un-numbered equation of page 29 should be $1/\sqrt{4s}$.

   (e) The line above (2.50) should be $t\,F(t) \simeq (4\pi t)^{-1/2}$, while Eq. (2.50) should be

   $$F(t) \simeq \frac{1}{\sqrt{4\pi}}\frac{1}{t^{3/2}} \quad \text{when} \quad t \to \infty.$$

   (f) In Eqs. (2.52), (2.53), and (2.54), there is no factor of 4.

   (g) The un-numbered equation after (2.56) should be

   $$F(s) \simeq [1 - (6P(0))^{-1}] - \frac{\sqrt{s}}{24\pi P(0)^2} = \mathcal{R} - \frac{3(1-\mathcal{R})^2\sqrt{s}}{2\pi} .$$

   In the following two sentences, the eventual return probability is $\mathcal{R} = [1 - (6P(0))^{-1}]$ and the comparison should be made with Eq. (2.48). Finally Eq. (2.57) should be

   $$F(t) \simeq \frac{3(1-\mathcal{R})^2}{4\pi^{3/2}}\frac{1}{t^{3/2}} .$$

3. Page 43, first line of Eq. (2.101). The second term on the right-hand side should read $\gamma^{-2}v_0^2(1-e^{-\gamma t})^2$. (Thanks to Nuno Araújo.)

4. Page 48, third line: "problem 2.32" (not 2.31).

5. Page 52, problem 2.3: The displayed equation should be replaced with

$$sP(n,s) - P(n,t=0) = P(n+1,s) + P(n-1,s) - 2P(n,s).$$

Furthermore, the correct expression for $P(n,s)$ is $P(n,s) = \lambda_-^{|n|}/(s+2-2\lambda_-)$ and $a = 1/(s+2)$. In this problem, it is implicitly assumed that the random walk starts at the origin. (Thanks to Ben Snow.)

6. Page 52, problem 2.6: The correct expression for the Laplace transform is

$$P_\pm(x,s) = \frac{1}{\sqrt{v^2+4Ds}}\, e^{\alpha_\mp x}.$$

7. Page 53, Problem 2.9: The constant C is missing the factor 2^μ. The correct expression is $C = -A\sqrt{\pi}\,\Gamma(-\mu/2)/\big[2^\mu\,\Gamma\big((1+\mu)/2\big)\big]$.

8. Page 56, problem 2.23, the boundary condition should read $c(r=a,t) = 0$. (Thanks to Colin Howard.)

Chapter 3:

1. Page 84, line 7: The sentence should read "Consequently, in the reference frame moving at the center of mass velocity, the particle velocities asymptotically decay as ..."

2. Equation (3.79): On the second line, the quantity T^{a_3/a_2} should be replaced with $\big(T/T(0)\big)^{a_3/a_2}$. Similarly, on the third line, the quantity T^{a_4/a_2} should be replaced with $\big(T/T(0)\big)^{a_4/a_2}$.

Chapter 4

1. Page 129, 2 lines above Eq. (4.61), the inequality should read $j \gg 1$. (Thanks to Alexander Povolotsky.)

Chapter 5:

1. Page 166, footnote 17. The words "that is smaller" should be replaced by "not greater". Also in this footnote, the equation cited should be (5.102).

2. Page 169, problem 5.4. The second term on the right-hand side of the master equation should be $c_k(kM_0+1)$. (Thanks to Ed Reznik.)

Chapter 6

1. Page 174, the second line after Eq. (6.2) should read: "... while the second term accounts for the creation of a fragment of size x due to the breakup of a larger cluster of size y. (Thanks to Alexander Povolotsky.)

2. Page 179, Eq. (6.21): Replace $M_k(s)$ with $c_k(s)$.

Chapter 7:

1. Page 205, paragraph 3 lines after Eq. (7.13): replace "'fragmentation' of an $(x+1)$-void by ..." with "'fragmentation' of an y-void, with $y \geq (x+1)$, by ...". (Thanks to Alexander Povolotsky.)
2. Page 223, two lines above Eq. (7.61). The equation reference should be to (7.60) not (7.61).

Chapter 8:

1. Page 240, 5 lines from the bottom and also the last line: replace Section 2.5 with Section 2.7. (Thanks to Alexander Povolotsky.)
2. Page 243, unnumbered equation below (8.23), second line should be

$$\frac{\sqrt{6}}{64\,\pi^3}\,\Gamma\left(\frac{1}{24}\right)\Gamma\left(\frac{5}{24}\right)\Gamma\left(\frac{7}{24}\right)\Gamma\left(\frac{11}{24}\right).$$

3. Page 251, Eq. (8.54). Here σ_m is the initial spin value at site m.
4. Page 252, 8 lines after Eq. (8.56) the text should read: "...satisfies a specified initial condition $G_k(t=0)$ and the...". Then in the first line of Eq. (8.57), $G_\ell(0)$ should be replaced by an arbitrary set of constants, say A_ℓ. Then in lines 2 and 3 of this equation $G_\ell(0)$ refers to the specified arbitrary initial condition.
5. Page 274, problem 8.3(c) should read: You may need to compute integrals of the form

$$\int_0^\infty e^{-x}\,[I_0(x) - I_1(x)]\,dx \qquad \text{and} \qquad \int_0^\infty e^{-2x}\,I_0(x)\,[I_0(x) - I_1(x)]\,dx\,.$$

   The three integrals given in the text are divergent, but their differences given above are convergent.

Chapter 9:

1. Page 317, first line. Replace "n exclusion points" with "n exclusion zones". (Thanks to Alexander Povolotsky.)

   Page 302, problem 9.19(e). The text should read: "Let $g(x,t)$ be the density of islands ...".

Chapter 10:

1. Page 331, the inline equation just above Eq. (10.20) should read: $3\psi(x)\left(\int_x^\infty \psi(y)\,dy\right)^2$. Then in Eq. (10.20), the factor $\frac{1}{3}$ on the right-hand side should not appear.
2. Page 341, long displayed equation inside the box: In the second line, the factor $\exp[N(F(x_0)]$ should be $\exp[NF(x_0)]$. In the third line, $\simeq$ should be $=$ and the factor $\exp[N|F(x_0)|]$ should be $\exp[NF(x_0)]$. (The absolute value is superfluous.)

3. Page 334, problem 10.2. The metastable-state energies are incorrect. For the uniform distribution the energy should be $\mathcal{E}_{\mathcal{M}} = -\frac{5}{12}$ (instead of $-\frac{17}{36}$). For the exponential distribution the energy should be $\mathcal{E}_{\mathcal{M}} = -\frac{8}{9}$ (instead of $-\frac{26}{27}$).

Chapter 12:

1. Page 376: The first eigenvalue for the fourth fixed point is $\lambda_1 = -1$. (Thanks to Nuno Araújo.)
2. Page 385: The subsection title should be "Bimolecular reaction" not "Biomolecular reaction". (Thanks to Alexander Povolotsky.)
3. Page 393: The second term inside the first bracket on the right-hand side should be $\frac{2}{V}$. (Thanks to Naoki Masuda.)
4. Page 396, line 8: The reference should be to problem 2.20.

Chapter 13:

1. Pages 423–4: Second line from the bottom on page 423, the formula should read $[Q^k_{n+1} - \widehat{Q}^k_n]/2$. On page 424, line 5, the formula should read $[Q^k_n - \widehat{Q}^k_n]/2$.
2. Page 438, exercise 13.7. The text of this problem should be updated as follows: Consider aggregation with a spatially localized source. Complete the discussion in the text for general spatial dimension and show that the total number of clusters asymptotically grows as

$$\mathcal{N} \sim \begin{cases} \ln t & d = 1 \\ (\ln t)^2 & d = d_c = 2 \\ \sqrt{t} & d = 3 \\ t/\ln t & d = d^c = 4 \\ t & d > 4 \end{cases}$$

It further follows from (13.84) that the stationary cluster density is given by

$$N \sim \begin{cases} r^{-1} & d = 1 \\ r^{-2}\ln r & d = d_c = 2 \\ r^{-2} & d = 3 \\ r^{-2}(\ln r)^{-1} & d = d^c = 4 \\ r^{-(d-2)} & d > 4 \end{cases}$$

In one and two dimensions, one cannot use the reaction-diffusion equation approach, Eq. (13.74), to establish these results. Instead, try to derive these behaviors heuristically using the results from diffusion-controlled aggregation in low spatial dimensions.

Hint: Argue that the naive reaction term, N^2 in (13.74), should be replaced by $N^2/\ln(1/N)$ in two dimensions and by N^3 in one dimension. Then repeat the analysis in the main text and deduce $N \sim r^{-2}\ln(r)$ in two dimensions and $N \sim r^{-1}$ in one dimension. Notice that the asymptotic $\mathcal{N} \sim (\ln t)^2$ differs from the (erroneous!) result in the top line of (13.85) which was derived using (13.75b) in two dimensions, $N \sim r^{-2}$.

3. Page 393: The second term inside the first bracket on the right-hand side of Eq. (12.50) should be $\frac{2}{V}$. (Thanks to Naoki Masuda.)
4. Page 396, line 8: The reference should be to problem 2.20.

Chapter 14:

1. Page 453, Eq. (14.27). The correct form of the right-hand side of this equation is

$$\frac{N}{2} + \frac{1}{N-1}$$