Universality classes of foraging with resource renewal

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We determine the impact of resource renewal on the lifetime of a forager that depletes its environment and starves if it wanders too long without eating. In the framework of a minimal starving random-walk model with resource renewal, there are three universal classes of behavior as a function of the renewal time. For sufficiently rapid renewal, foragers are immortal, while foragers have a finite lifetime otherwise. In the specific case of one dimension, there is a third regime, for sufficiently slow renewal, in which the lifetime of the forager is independent of the renewal time. We outline an enumeration method to determine the mean lifetime of the forager in the mortal regime.

I. INTRODUCTION

What is the impact of renewal of resources on the state of a forager? If the environment is harsh and resources regenerate slowly, foragers may be confronted by perpetual scarcity. Thus a forager may often go hungry or even starve. Conversely, in an abundant environment where resources are quickly replenished, a forager may never experience starvation risk. Our goal is to map out the states of a forager as a function of the renewal time within the framework of the minimal "starving random-walk" model [1] that we define below.

The random-walk model has been frequently invoked to describe the motion of foraging animals [2–4], as well as a wide variety of classic applications [5–9]. In the wild, foraging animals can die from many causes, such as diseases or old age [10]. The underlying dynamics from these causes of death can be described by a "mortal" random walk that dies according to a specified lifetime distribution [11–13]. This model also describes a variety of applications to diverse fields, such as the diffusion of light in human tissue [14], and biologically inspired search problems [15]. In the context of foraging, an important contributor to mortality is the possibility that a forager is unsuccessful in its search for food within its habitat [16]. Thus the age at which a forager dies is also determined by its trajectory and the amount of available resources. This coupling between the lifetime of a living organism and its trajectory, along which environmental resources are depleted, defines a nontrivial class of random-walk problems [17–22]—including the starving random-walk model—that are relatively unexplored.

In the original starving random-walk model [1], a random walk irreversibly depletes its environment during its wanderings, and starves if it wanders too long in a resource-depleted region. Initially, each lattice site contains one food unit. Whenever a forager, which undergoes a random walk, arrives at a full site, the food is completely eaten. Once depleted, a site remains empty. Whenever the forager arrives at an empty site, it does not eat. If the forager wanders for $\mathcal{S}$ steps without eating, it starves. We can think of $\mathcal{S}$ as the metabolic capacity of the forager—the amount of time it can live without food before starving. Asymptotic expressions for the walker lifetime and the territory visited at the starvation time were

given in one dimension [1]. Estimates for these two quantities in dimensions $d \geqslant 3$ and a lower bound for the territory visited at starvation in $d = 2$ were also given. These results provide a first step in quantifying the interplay between the trajectory of a forager and the consumption of food and their effect on the lifetime of the forager.

However, the natural resources being consumed—preys, plants, water, and nutrients—typically obey their own dynamics. In particular, consumed resources usually do not disappear permanently [23,24]. Instead, they typically regenerate a certain time after they have been depleted [25–27]. This basic fact motivates our study of starving random walks with the possibility of renewal of resources (Fig. 1).

As we will discuss, resource renewal substantially modifies the properties of a starving random walk. Using extreme trajectory arguments, we will argue that the correlations induced by the coupling between the trajectory of a forager, its metabolic capacity $\mathcal{S}$, and the dynamics of the resources lead to three universal regimes of behavior that are determined by $\mathcal{S}$ and the bounds $\mathcal{R}_1$ and $\mathcal{R}_2$ of the support of the renewal time distribution, but are insensitive to the shape of this distribution (Fig. 2). We will first demonstrate the existence of a transition between an immortal regime, in which the forager cannot starve, no matter what its trajectory, and a mortal regime, where the forager must eventually starve. Both regimes arise
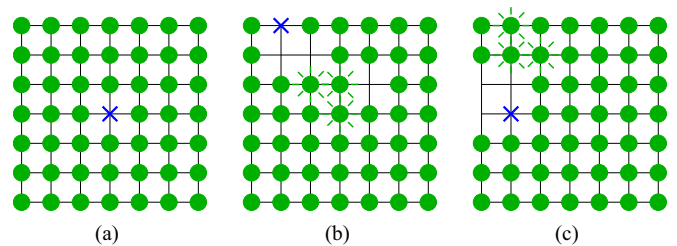

FIG. 1. Starving random walk with probabilistic resource renewal in two dimensions: (a) initial state and later times (b) and (c). Each site initially contains food that is eaten when found by the forager. Food reappears on empty sites after a random renewal time (represented here as glowing circles). The forager starves if it wanders $\mathcal{S}$ steps without eating.
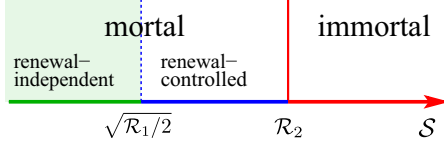
FIG. 2. Model phase diagram. The shaded zone, where the lifetime is independent of renewal rate, occurs only for $d = 1$; the two other regimes arise for any d. The thresholds between these regimes are given by Eqs. (1) and (6).
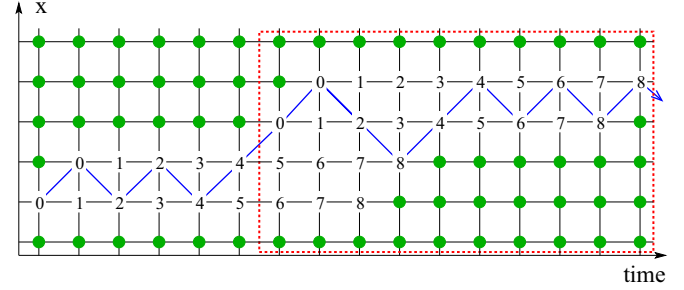

FIG. 3. Illustration of the common pattern (inside the dashed rectangle) of all extremal trajectories for the case of renewal time $\mathcal{R} = 9$. This pattern consists of depleting two consecutive sites at one end of the desert. The forager wanders inside this desert which gradually shortens until it reaches length 2. The pattern ends when the walker is certain to land on a food-containing site. Green dots denote food-containing sites and the numbers give the ages of each site.

in any spatial dimension. We will also show that a third regime arises in one dimension only, in which renewal is so slow that the forager lifetime is the same as in the case of no renewal. We also develop an enumeration method that yields, in principle, the exact value of the mean lifetime of a forager in the mortal regime.

We will first address the case of starving random walks with deterministic renewal in one dimension (Sec. II). In Sec. III, we extend our basic results to the case of probabilistic renewal and to higher dimensions. Some brief conclusions are given in Sec. IV.

II. DETERMINISTIC RENEWAL IN ONE DIMENSION

Let us first investigate starving random walks with *deterministic* renewal in one dimension. Such a deterministic mechanism describes plants that grow at a fixed rate to reach an edible size a fixed time after having been previously defoliated [28]. We posit that food that is eaten at time step t reappears at time $t + \mathcal{R}$, with $\mathcal{R}$ an integer. In each time step, the time elapsed since an empty site was depleted is increased by one and food appears at any site where this time equals $\mathcal{R}$. As part of this same time step, the walker hops to one of its nearest neighbors. If the site contains food, which may have appeared just before the walker arrives, the food is eaten. We call the set of empty sites, which may or may not be connected, the "desert."

A. Immortality

For a forager with metabolic capacity $\mathcal{S} \in \mathbb{N}$, we now determine the range of the renewal times for which immortality arises. A forager is immortal if it survives forever on any trajectory and, in particular, on the most unfavorable trajectories. The set of such trajectories is infinite, but they all possess the common pattern (Fig. 3) that the walker depletes two sites in a row at one end of the desert, before wandering within the desert as long as possible until being certain, because of resource renewal, to land on a food-containing site (see Appendix A for details). It takes $\mathcal{R}$ time steps before food reappears on one of these two sites (Fig. 3). Thus, roughly speaking, when $\mathcal{R} < \mathcal{S}$, the forager necessarily survives. The precise criterion actually is $\mathcal{R} \leqslant \mathcal{R}^*$, with

$$\mathcal{R}^* = \begin{cases} \mathcal{S}, & \mathcal{S} \text{ even} \\ \mathcal{S} + 1, & \mathcal{S} \text{ odd,} \end{cases} \tag{1}$$

due to the even-odd oscillations of a nearest-neighbor random walk (Appendix A).

B. Mean lifetime in the mortal regime

On the other hand, when $\mathcal{R} > \mathcal{R}^*$, the forager is mortal and thus eventually starves with probability 1 because we can construct trajectories that necessarily lead to starvation. These trajectories again involve the forager first carving out a desert and then wandering strictly within this desert so that renewal does not reach the forager before it starves. In contrast to the situation in Fig. 1, the walker starves before it reaches a site where the resource has been regenerated. By their very existence, such trajectories are achieved with nonzero probability. From classical results about Markov chains (see, e.g., [29]), every trajectory will eventually generate a configuration for which the forager starves. Thus there are two regimes of behavior—immortality and mortality. The boundaries between these regimes depend only on the metabolic capacity $\mathcal{S}$ of the forager and the renewal time $\mathcal{R}$.

Using the Markov chain formalism, we can, in principle, determine the mean lifetime of the forager by enumerating the configurations of the system as the forager wanders. A configuration is defined as the location and age of each empty site in the desert, the position of the forager in the desert, and the number of time steps elapsed since the forager last ate. Here the age of an empty site is the time since the food was last consumed at this site. Thus a newly empty site has age 0, while a site that will regenerate at the next step has age $\mathcal{R} - 1$. Because the desert has a finite size, the number of configurations is finite. We can therefore write the transition matrix that describes the evolution of the system at each step of the forager and thereby extract its mean lifetime.

Let us illustrate this enumeration for the simple case of $\mathcal{S} = 2$ and $\mathcal{R} = 3$. For this example, there are five distinct configurations (Fig. 4). State ① arises after the first step, and the evolution of the system from one state to another is shown in the figure. The associated transition matrix is

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 1/2 & 0 & 0 & 1/2 & 0 \\ 0 & 1/2 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 1/2 & 0 \\ 0 & 1/2 & 0 & 0 & 1 \end{pmatrix} \equiv \begin{pmatrix} Q & 0 \\ V & I \end{pmatrix}, \tag{2}$$
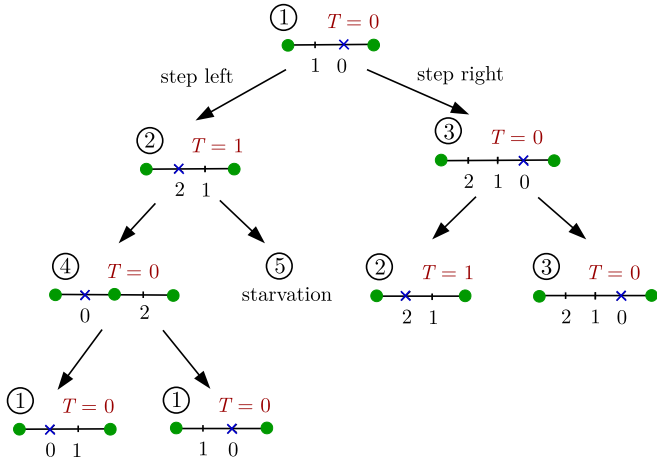
FIG. 4. State space of the system for metabolic capacity $\mathcal{S} = 2$ and renewal time $\mathcal{R} = 3$. The circled numbers denote the distinct states and T is the time elapsed since the forager last ate.

where the states are listed in order ①−⑤, with

$$Q = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1/2 & 0 & 0 & 1/2 \\ 0 & 1/2 & 0 & 0 \\ 1/2 & 0 & 0 & 1/2 \end{pmatrix}, \qquad (3)$$

and $V = (0, \frac{1}{2}, 0, 0)$. We define the matrix (see [29])

$$N \equiv (I - Q^+)^{-1} = \begin{pmatrix} 2 & 2 & 1 & 2 \\ 1 & 2 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 1 & 2 & 1 & 3 \end{pmatrix}, \qquad (4)$$

where Q^+ is the transpose of Q. Each entry N_{ij} in this matrix is the average time that a system, which ultimately reaches starvation, spends in configuration j when it starts from configuration i [29]. From this matrix we can extract the mean absorption time t_i starting from the state i. These are given by

$$\begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \end{pmatrix} = N \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 7 \\ 5 \\ 8 \\ 7 \end{pmatrix}. \qquad (5)$$

Thus the mean lifetime of the forager is $t_1 + 1 = 8$, because after the first step, the system is necessarily in the state ①. In practice, this approach quickly becomes intractable because the number of possible configurations becomes prohibitively large when both $\mathcal{R}$ and $\mathcal{S}$ are large. Nevertheless, this approach gives a well-defined prescription for computing the average time until the forager starves.

C. Renewal independent regime

In one dimension, the mortal regime can be further divided in two subregimes: (a) forager lifetime dependent on $\mathcal{R}$, and (b) renewal independence—lifetime independent of $\mathcal{R}$. Clearly, as $\mathcal{R}$ increases, the forager lifetime decreases and approaches the no-renewal limiting value as $\mathcal{R} \to \infty$. Does this decrease stop when $\mathcal{R}$ reaches a finite critical value $\mathcal{R}^\dagger$, or does the decrease continue as $\mathcal{R} \to \infty$? To resolve this question, we determine
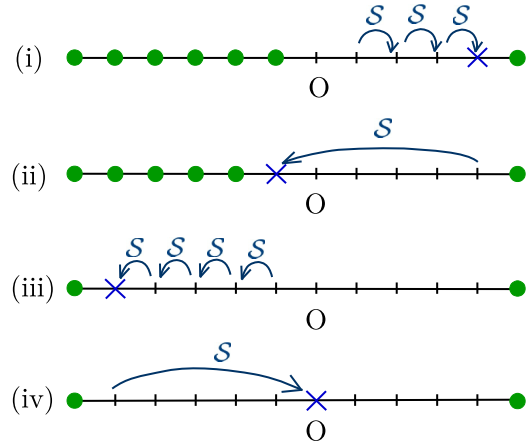

FIG. 5. Optimal trajectory for a forager to live the longest and still be able to return the origin O and consume the replenished resource on this site, for the case $\mathcal{S} = 5$. (i) The forager eats a site on the right edge of the desert every $\mathcal{S}$ steps until carving the largest semidesert that it can cross. (ii) The forager crosses the desert. (iii) Reflection of stage (i). (iv) The forager crosses the left semidesert to reach the origin. Such an excursion lasts $\mathcal{R}^\dagger$ steps [Eq. (6)].

if there is at least one trajectory for which the forager can return to a replenished site without starving. If there is such a trajectory, then renewal is relevant, as the lifetime of the forager depends on the renewal time.

For a forager to return to a site on which food is renewed requires: (i) living long enough for such a renewal to occur and (ii) staying sufficiently close to this site to reach it without starving. These two attributes are most easily satisfied for a site (which we take as the origin) at which food has just been eaten and is surrounded by full sites. We determine the largest value of $\mathcal{R}$ for which the forager can return to this origin after the food at this site has been renewed.

To return to the origin without starving imposes the constraint that the forager does not stray more than $\mathcal{S}$ steps from the origin. Moreover, to maximize the time that the forager wanders, it should eat only when it really needs to, that is, every $\mathcal{S}$ steps. The trajectory on which the forager lives the longest while staying within $\mathcal{S}$ steps of the origin therefore consists of the following components (Fig. 5): (i) The forager creates a semidesert of length $\mathcal{S}$ on one side, say to the right, of the origin. During this creation of the semidesert, the forager eats by moving to a previously unvisited site only every $\mathcal{S}$ steps. (ii) The forager successfully crosses this semidesert, which is the longest possible for which a successful traversal is possible. (iii) The forager creates a mirror image semidesert of length $\mathcal{S}$ to the left of the origin. (iv) The forager crosses this left semidesert and fetches the regenerated food at the origin.

The duration of this excursion is roughly $2\mathcal{S}^2$, as the forager has eaten $2\mathcal{S}$ times when it returns to the origin. The enumeration of the above sequence of moves (Appendix B) gives the maximal renewal time $\mathcal{R}^\dagger$ with

$$\mathcal{R}^\dagger = \begin{cases} 2\mathcal{S}^2 - 3\mathcal{S} + 4, & \mathcal{S} \text{ even} \\ 2\mathcal{S}^2 - \mathcal{S} + 1, & \mathcal{S} \text{ odd}. \end{cases} \qquad (6)$$

For $\mathcal{R} \leqslant \mathcal{R}^\dagger$, a forager has a nonzero probability to eat food at a site where renewal has occurred. Conversely, for $\mathcal{R} > \mathcal{R}^\dagger$, a forager cannot reach any replenished site, so that renewal has no impact on the forager lifetime. Hence the mean lifetime does not gradually converge to the limiting no-renewal value, but rather reaches this value for $\mathcal{R} = \mathcal{R}^\dagger + 1$. Thus we infer that there exists a renewal-independent regime (Fig. 2) for the lifetime of the forager.

III. EXTENSIONS

We now extend the starving random-walk model with resource renewal to accommodate two ecologically realistic features: (i) probabilistic renewal, in which the resource is regenerated at a random time after depletion, rather than after a fixed time $\mathcal{R}$, and (ii) starving random walks with resource renewal in higher dimensions.

A. Probabilistic renewal

Suppose that each empty site is replenished a time τ after the resource at that site has been consumed, with τ drawn from a continuous distribution with support $[\mathcal{R}_1, \mathcal{R}_2] \subset \mathbb{R}^+$. This means that, for a given site, renewal cannot happen before a time $\mathcal{R}_1$ and also that replenishment necessarily occurs within a time $\mathcal{R}_2$ after depletion. We make no assumption on the shape of this distribution. In particular, $\mathcal{R}_1$ can be zero and $\mathcal{R}_2$ can be infinite.

In the case of deterministic renewal (Sec. II), we saw that the random walker is immortal when it is certain to land on a food-containing site before starving, even on the most unfavorable trajectories. The criterion for immortality in this case is determined by $\mathcal{R} \leqslant \mathcal{R}^*$, with $\mathcal{R}^*$ given in Eq. (1). For probabilistic renewal, the walker is sure to land on a food-containing site before starving if every renewal time in the support of the renewal-time distribution is smaller than $\mathcal{R}^*$. Therefore, immortality is assured when $\mathcal{R}_2 \leqslant \mathcal{R}^*$. However, if the upper bound $\mathcal{R}_2$ is infinite, that is, if replenishment on some sites can take an arbitrarily long time, immortality cannot occur.

On the other hand, if $\mathcal{R}_2 > \mathcal{R}^*$, there exist patterns of steps for the random walker that lead to starvation, as in the case of deterministic renewal. Hence the walker is mortal. Additionally, the enumeration method presented above for deterministic renewal can be implemented in a similar manner for the case of probabilistic renewal. In the probabilistic case, however, food does not reappear at a fixed time after depletion but at a time that is drawn from the renewal time distribution. What this means practically is that the number of configurations in probabilistic renewal is larger than that for deterministic renewal. Moreover, if the support of the distribution of renewal times is unbounded, the enumeration approach fails because the number of configurations is infinite.

We also argued in Sec. II that there exists a second transition inside the mortal regime between a subregime in which dynamics of the renewal controls the lifetime of the walker and a subregime where the lifetime becomes independent of the renewal dynamics. We inferred the existence of this transition by constructing the extremal trajectory that demarcates this second transition (Fig. 5). If renewal has not occurred at the

origin when the walker reaches this site at the end of the pattern of steps of Fig. 5, then renewal has no impact on the trajectory. Thus, the renewal-independent regime arises if every depleted site remains empty for at least $\mathcal{R}^\dagger$ steps. In the case of probabilistic renewal, this second transition occurs when $\mathcal{R}_1 > \mathcal{R}^\dagger$.

These results are summarized in the phase diagram of Fig. 2; this is our key result.

B. Higher dimensions

We now turn to starving random walks on higher-dimensional lattices for the general situation of probabilistic renewal. The class of trajectories that are the least favorable for the survival of the walker (see Fig. 3), still arises in higher dimensions. Hence the immortality criterion $\mathcal{R}_2 \leqslant \mathcal{R}^*$ derived in the previous subsection remains valid, independent of the spatial dimension. Moreover, in the mortal regime, the enumeration method still works and can be used to determine the mean lifetime of a random walker.

In contrast to the transition to immortality, a transition to a renewal-independent regime does not occur in higher dimensions. The unique feature of one dimension is that the walker *must* traverse the desert that was carved by its previous trajectory to reach replenished sites. In contrast, in higher dimensions, there always exist trajectories on which a forager can stay alive for an arbitrarily long time and still return to the replenished sites without starving, because it can avoid the desert instead of having to cross it. Thus the renewal time—no matter how long—always affects the lifetime of the forager in greater than one dimension. We conclude that in higher dimensions there is only an immortal regime and a mortal regime in which the lifetime is a function of the renewal dynamics. The transition between these two regimes is determined only by the upper bound of the distribution of renewal times, and not by the shape of this distribution, or by the spatial dimension.

IV. SUMMARY AND CONCLUSION

To summarize, the renewal of resources has a dramatic effect on starving random walks. There exist three regimes of behavior as a function of the renewal time $\mathcal{R}$: (i) an immortal regime where a forager can live forever, (ii) a mortal regime where the forager lifetime is finite and depends on $\mathcal{R}$, and (iii) a renewal-independent mortal regime where renewal does not affect the lifetime of a forager. The latter arises only in one dimension; here, the average forager lifetime equals the value obtained in the absence of any renewal. In contrast, regimes (i) and (ii) arise in any spatial dimension and are universal with respect to the distribution of renewal times. The transitions between these regimes depend only on the bounds of the support of the renewal-time distribution and not on its shape. Much of this new phenomenology is controlled by the *times between visits* to distinct sites in a random walk, an apparently unexplored feature of site visitation statistics of random walks. Finally, we outlined an enumeration method to determine the mean forager lifetime in the mortal regimes (ii) and (iii). Average values of other basic observables, such as the number of distinct sites visited at starvation, the number of

sites in the desert, and the time spent in a certain configuration, can also be extracted from this approach.

Immortality is the main new feature that arises as a result of resource renewal. If a wandering organism can survive without food longer than the time needed for resources to be replenished, its lifetime is no longer limited by starvation but rather by external constraints (such as predation, diseases, life expectancy of the species, etc.). We speculate that perhaps the metabolic capacity of a given species is determined by the characteristic time for renewal of resources.

This work represents a first step to provide insight into the impact of resource renewal on the fate of a forager that depletes its environment by consumption. While most of our qualitative analysis was specific to the case of one dimension, our approach applies for any spatial dimension and also to arbitrary renewal dynamics. A basic question that we have not fully addressed is the analytic determination of the mean lifetime of the walker in the mortal regime. This calculation is of particular importance in two dimensions where it should be directly applicable to the modeling of animal behavior. In addition to developing a more complete theory in two dimensions, the inclusion of additional realistic features to this model, such as sensory awareness of the forager, or interactions between several foragers, such as sharing resources, are needed to give a more realistic description of ecosystems.

ACKNOWLEDGMENTS

APPENDIX A: THE BOUND $\mathcal{R}^*$

We provide the details for the determination of Eq. (1) in the case of deterministic renewal. This bound $\mathcal{R}^*$ depends on the parity of the metabolic capacity $\mathcal{S}$ because of the even-odd oscillations of the nearest-neighbor random walk. As described in the main text, the most unfavorable trajectories—on which the walker remains the longest without eating—possess the common pattern of eating two consecutive sites at one edge of the desert and then wandering as long as possible inside the desert, without depleting any additional site. This last feature implies that the desert gradually shortens as renewal happens, finally confining these most unfavorable trajectories to a two-site desert, made of the first two depleted sites of the pattern (see Fig. 6).

Depending on the value of $\mathcal{S}$, the walker either survives long enough to eat when renewal happens on these two sites (in the immortal regime), or starves (in the mortal regime). We determine the maximal value of the renewal time $\mathcal{R}^*$ that corresponds to the immortal regime for an example of most unfavorable trajectory for two consecutive values of $\mathcal{S}$ (Fig. 7). The walker is sure to eat before starving when the renewal time is $\mathcal{R} = \mathcal{R}^*$, even on this unfavorable trajectory (left column of Fig. 7), but can die if $\mathcal{R} = \mathcal{R}^* + 1$ (right column of Fig. 7). For this example, which can be generalized to every value of $\mathcal{S}$, we see that

$$\mathcal{R}^* = \begin{cases} \mathcal{S}, & \mathcal{S} \text{ even} \\ \mathcal{S} + 1, & \mathcal{S} \text{ odd.} \end{cases} \tag{A1}$$
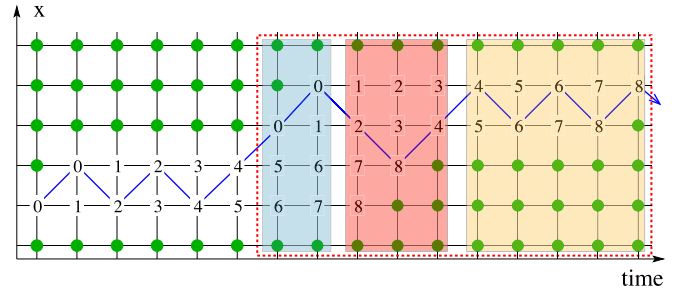

FIG. 6. More detail of the common pattern (inside the dashed rectangle) of all extremal trajectories for renewal time $\mathcal{R} = 9$. This pattern starts by depleting two consecutive sites at one end of the desert (shaded in blue). The forager then wanders inside this desert which gradually shortens (shaded in red) until it reaches length 2 (shaded in yellow). The pattern ends when the walker is sure to land on a food-containing site. Green dots denote food-containing sites and the numbers give the ages of each site.

APPENDIX B: THE BOUND $\mathcal{R}^\dagger$

We now give the details to derive Eq. (6) for the case of deterministic renewal. Note that the bound $\mathcal{R}^\dagger$ also depends on the parity of the metabolic capacity $\mathcal{S}$ because of the even-odd oscillations of the nearest-neighbor random walk. If renewal is sufficiently quick, there exist trajectories for which the walker can return to a replenished site, in particular the origin of the walk, before starving. On the other hand, if renewal is too slow,
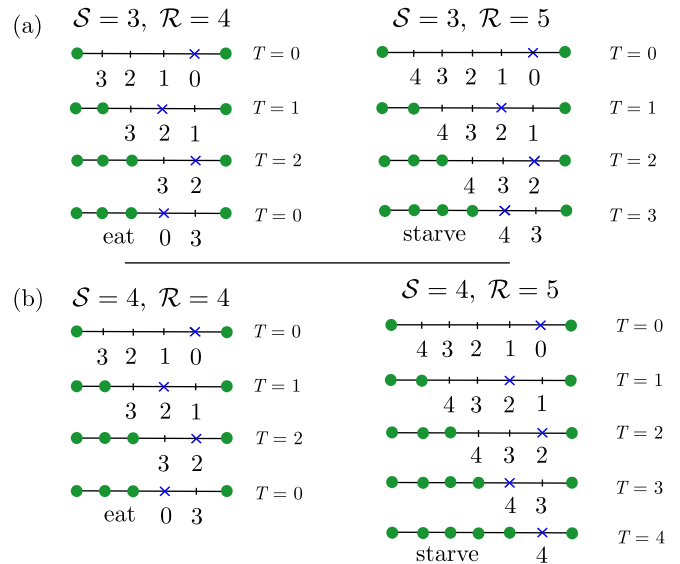

FIG. 7. Destiny of the walker on the most unfavorable trajectories, for (left column) the maximal renewal time $\mathcal{R}^*$ that leads to immortality, and (right column) for the minimal value of $\mathcal{R}$ that leads to mortality. The cases of odd and even metabolic capacities $\mathcal{S}$ are shown in (a) and (b), respectively. For both even and odd $\mathcal{S}$, the maximal value of $\mathcal{R}$ that yields immortality is the smallest even integer equal to or greater than $\mathcal{S}$. Here T denotes the number of steps since the last meal. The dots represent food and the cross indicates the walker. Empty sites are labeled by the time elapsed since the food on these sites was eaten.

then the walker either dies before this renewal happens, or carves a desert that is too large to be crossed without starving.

As mentioned in the main text, the maximal value $\mathcal{R}^\dagger$ of the renewal time for which the walker has a chance to return to the origin after the resource on this site has been renewed requires that (i) the walker lives long enough for this renewal to occur, and (ii) the walker must stay sufficiently close to the origin to be able to reach it without starving. The walker can satisfy these two constraints by eating approximately every $\mathcal{S}$ steps and by staying within a segment of size $2\mathcal{S}$ centered on the origin.

The trajectories on which the walker lives the longest while remaining within $\mathcal{S}$ steps of the origin consist of the following (Fig. 3): (i) The walker creates a desert of $\mathcal{S}$ sites on one side of the origin, say the right. During this creation of this desert, the walker waits as long as possible between each meal, that is, $\mathcal{S}$ steps if $\mathcal{S}$ is odd, or $\mathcal{S} - 1$ steps if $\mathcal{S}$ is even. Indeed, starting from the right edge of the desert, the walker needs an even number of steps to come back to this edge; thus an odd number of steps is required to eat (and deplete) the resource

at the next site on the right side. (ii) The walker crosses the desert and reaches the site to the left of the origin after $\mathcal{S}$ steps. (iii) The walker creates a mirror image desert on the left side of the origin, by depleting $\mathcal{S} - 1$ new sites. (iv) The walker crosses the left desert and reaches the origin after $\mathcal{S}$ steps. For the maximal value $\mathcal{R}^\dagger$ of the renewal time, the walker finally reaches the origin at the end of stage (iv) at the time step where the origin regenerates.

We now determine $\mathcal{R}^\dagger$ by counting the number of steps on this trajectory. The walker eats the site on the right of the origin at time step 1, and then takes a time $\mathcal{S}(\mathcal{S} - 2)$ if $\mathcal{S}$ is odd, and $(\mathcal{S} - 1)(\mathcal{S} - 2)$ if $\mathcal{S}$ is even to complete phase (i). Similarly, phase (iii) lasts $\mathcal{S}(\mathcal{S} - 1)$ if $\mathcal{S}$ is odd and $(\mathcal{S} - 1)^2$ if $\mathcal{S}$ is even. Moreover, phases (ii) and (iv) both last $\mathcal{S}$ steps independent of the parity of $\mathcal{S}$. Assembling these results yields the critical value:

$$\mathcal{R}^\dagger = \begin{cases} 2\mathcal{S}^2 - 3\mathcal{S} + 4, & \mathcal{S} \text{ even} \\ 2\mathcal{S}^2 - \mathcal{S} + 1, & \mathcal{S} \text{ odd}. \end{cases} \tag{B1}$$

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