Erratum to J. Phys. A: Math. Gen. 38, 2555 (2005)

Let $\{X_n, n \in \mathbf{N}\}$ denote the excited random walk on the integers, with absorbing states at 0 and x. Let T_x be the average time to hit site x for the first time. Initially, the walk starts at site 1, and we assume that it eats all cookies between and including sites 1 and $x-2$. We also assume that the walk just stepped to site $x-1$ for the first time, and we are interested in the average time to step to site x for the first time, with the condition that the walk does not visit site 0. Note that the first step from site $x-1$ is biased, but after this step there are no cookies on sites $1, 2, \dots, x-1$, so the walk is unbiased on these sites. The conditional average time to first hit site x can be split up by the total probability rule

$$\begin{aligned} &E(T_x|X_0 = x-1, X_\infty = x) \\ &\quad = \sum_{y=x-2,\,x} E(T_x|X_1 = y, X_\infty = x)\, P(X_1 = y|X_0 = x-1, X_\infty = x\,, \end{aligned} \tag{1}$$

where X_∞ is the final absorbing state at 0 or x. The first factor can be simplified as

$$E(T_x|X_1 = y, X_\infty = x) = 1 + E(T_x|X_0 = y, X_\infty = x)$$

and for an unbiased random walk starting from site y, the conditional average to reach site x without hitting site 0 is

$$E(T_x|X_0 = y, X_\infty = x) = \tfrac{1}{3}(x^2 - y^2)\,.$$

Hence

$$\begin{aligned} E(T_x|X_1 = x-2, X_\infty = x) &= 1 + \tfrac{4}{3}(x-1) \\ E(T_x|X_1 = x, X_\infty = x) &= 1\,. \end{aligned} \tag{2}$$

The second factor in (1) simplifies in the usual way as

$$\begin{aligned} &P(X_1 = y|X_0 = x-1, X_\infty = x) \\ &\quad = \frac{P(X_1 = y, X_\infty = x|X_0 = x-1)}{P(X_\infty = x|X_0 = x-1)} \\ &\quad = \frac{P(X_\infty = x|X_1 = y, X_0 = x-1)P(X_1 = y|X_0 = x-1)}{P(X_\infty = x|X_0 = x-1)} \\ &\quad = \frac{P(X_\infty = x|X_0 = y)}{P(X_\infty = x|X_0 = x-1)} P(X_1 = y|X_0 = x-1)\,. \end{aligned} \tag{3}$$

The probability for an unbiased random walk to first hit the right boundary is given by

$$P(X_\infty = x|X_0 = x-2) = \frac{x-2}{x} \qquad\qquad P(X_\infty = x|X_0 = x) = 1.$$

However, when the walk starts from site $x-1$ we need to be careful, as the first step is still biased

$$\begin{aligned} P(X_\infty = x|X_0 = x-1) &= qP(X_\infty = x|X_0 = x-2) + pP(X_\infty = x|X_0 = x) \\ &= q\frac{x-2}{x} + p\,. \end{aligned} \tag{4}$$

Hence,

$$\begin{aligned} P(X_1 = x-2|X_0 = x-1, X_\infty = x) &= q\,\frac{(x-2)/x}{q(x-2)/x+p} = \frac{q\,(x-2)}{x-2q} \\ P(X_1 = x|X_0 = x-1, X_\infty = x) &= \frac{p}{q(x-2)/x+p} = \frac{px}{x-2q}\,. \end{aligned} \tag{5}$$

These two probabilities add up to 1, as they must.

Substituting these probabilities into our original expression (1) we obtain

$$E(T_x|X_0 = x-1, X_\infty = x) = 1 + \frac{4}{3}(x-1)\frac{q(x-2)}{x-2q} = 1 + \frac{4q}{3}\frac{(x-1)(x-2)}{x-2q}\,. \tag{6}$$

For $p = q = 1/2$ the cookies have no effect, and we recover the well known expression for the conditional escape time from an interval

$$E(T_x|X_0 = x-1, X_\infty = x) = \frac{2x-1}{3}$$

For the total time to reach site x from the initial site 1, we need to sum up the above expression

$$\begin{aligned} E(T_x|X_0 = 1, X_\infty = x) &= \sum_{n=2}^{x} E(T_n|X_0 = n-1, X_\infty = n) \\ &= x - 1 + \frac{4q}{3}\sum_{n=3}^{x}\frac{(n-1)(n-2)}{n-2q} \\ &= x - 1 + \frac{4q}{3}\sum_{n=2}^{x}\frac{n(n-1)}{n-1+2p}\,, \end{aligned} \tag{7}$$

which can be expressed in terms of the digamma function as

$$x - 1 + \frac{4q}{3}\left\{\frac{x(x+1)}{2} - 1 - 2p(x-1) + 2p(2p-1)\big[\Psi_0(x+2p) - \Psi_0(1+2p)\big]\right\}.$$

Its large x-asymptotic follows from

$$E(T_x|X_0 = x-1, X_\infty = x) \sim \frac{4q}{3}x\,. \tag{8}$$

Hence the total time scales as $2q\,x^2/3$, so that the reported behavior in the article, namely, that the total times is proportional to x^2, was correct.