Optimally frugal foraging

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We introduce the *frugal foraging* model in which a forager performs a discrete-time random walk on a lattice in which each site initially contains $\mathcal{S}$ food units. The forager metabolizes one unit of food at each step and starves to death when it last ate $\mathcal{S}$ steps in the past. Whenever the forager eats, it consumes all food at its current site and this site remains empty forever (no food replenishment). The crucial property of the forager is that it is *frugal* and eats only when encountering food within at most k steps of starvation. We compute the average lifetime analytically as a function of the frugality threshold and show that there exists an optimal strategy, namely, an optimal frugality threshold k^* that maximizes the forager lifetime.

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I. INTRODUCTION

Foraging is a fundamental ecological process whose phenomenology has been the basis for a large body of research (see, e.g., Refs. [1–6]). Theories of foraging have attempted to determine good strategies for a forager to maximize its food consumption. Such strategies balance the interplay between exploitation, where a forager consumes food in a current search domain, and exploration, where a forager moves to a new search domain that may be potentially richer. This dichotomy between exploitation and exploration underlies a wide range of optimization phenomena for which statistical physics ideas have been fruitful, including the management of firms [7,8], the multiarm bandit problem [9,10], the secretary problem [11], Feynman's restaurant problem [12], and human memory [13,14]. In each of these cases, there is a continual tension between continuing to exploit a current strategy or exploring a new part of the strategy space where resources may be more plentiful.

Typically these optimization problems do not account for the depletion of resources due to their consumption by the forager. The *starving random walk* [15–17] explicitly accounts for this basic coupling between forager motion and depletion. In this starving random walk model, a forager performs a random walk on a lattice in which each site initially contains $\mathcal{S}$ food units. When a forager lands on a food-containing site, all the food there is consumed and the forager is fully satiated. The forager metabolizes one unit of food at each step so that it starves when it last ate $\mathcal{S}$ time steps in the past. When the forager lands on an empty site, it comes one time unit closer to starvation. In this model, food is not replenished, so that the forager is doomed to eventually starve to death. One interesting aspect of the starving random walk is the nontrivial dependence of the forager lifetime on its intrinsic starvation time $\mathcal{S}$ and the spatial dimension d [15,16].

A basic feature of the starving random walk is that the forager mindlessly eats whenever food is encountered. Is it possible that the forager can live longer with a different consumption strategy? By incorporating the attribute of *frugality*, in which the forager eats only when it is nutritionally depleted below a specified level (Fig. 1), we will show that the average lifetime of a forager can be dramatically increased compared to the starving random walk. This frugality mimics what occurs in ecological foraging, where foragers reduce their activity when satiated and resume foraging only when sufficiently depleted; parallel behavior occurs in predatory animals [18–27]. Because of satiation, such animals do not eat all the food that they encounter [28]. The concept of "laziness" has also been invoked to describe how animals budget their time among different basic everyday activities [29]. For instance, ants have been reported to increase their foraging activity when they get close to starvation [30]. Existing random search models do not address such aspects of foraging. While delaying consumption might seem to be a risky survival strategy, we will show here that (i) frugality typically increases the forager lifetime and (ii) the lifetime is maximized at an optimal frugality.

II. THE MODEL

The frugal forager starts at the origin in a food paradise, where each lattice site initially contains $\mathcal{S}$ units of food. The forager immediately eats all the food at the origin, so it begins in a fully satiated state. Subsequently, the forager performs a lattice random walk, independent of whether the forager lands on a food-containing site or an empty site. That is, the forager can only detect food that is on the same site as the forager.

The forager is characterized by its metabolic capacity, or intrinsic starvation time $\mathcal{S}$, which is the number of random-walk steps that the forager can travel without encountering food before starving to death. In each step, the forager metabolizes one unit of food. If the forager lands on an empty site, the forager therefore comes one time unit closer to starvation. If the forager lands on a food-containing site *and* the forager is

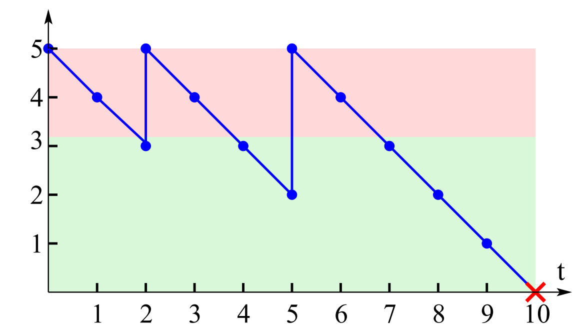

FIG. 1. The nutritional state of a frugal forager as a function of time for the case $\mathcal{S} = 5$ and frugality threshold $k = 3$. In the red zone, the forager does not eat, even when it encounters food, while in the green zone the forager eats whenever it encounters food. This forager starves at $t = 10$.

within k time units of starvation, then all the food at this site is consumed and the forager returns to a fully satiated state.

The limiting situation where the forager always eats when it encounters food is known as the starving random walk [15,16], which here we term as the normal forager. We denote k as the *frugality threshold*, which can lie in the range $0 \leqslant k \leqslant \mathcal{S} - 1$ (Fig. 1). The case of $k = \mathcal{S} - 1$ corresponds to the normal forager that eats anytime it encounters food. Because the the forager returns to a fully satiated state upon eating, up to k food units are "wasted" by this consumption rule. The opposite limiting case of $k = 0$ corresponds to the maximally frugal forager that will be discussed in Sec. IV A. Figuratively, this frugal consumption rule is akin to a decision that we often make when driving a long distance on a highway, where we fill the gas tank of the car only when the fuel level falls below a specified level.

As in the case of the normal forager, an important feature of the dynamics is that the mortality of the frugal forager is coupled to its gradually (and quite slowly) depleting environment. Moreover, the lifetime of the forager depends on its full trajectory. That is, this foraging process is inherently non-Markovian because the dynamics depends on the times of all the forager's previous encounters with food, i.e., the times between visits to distinct sites of a random walk [31–34].

III. SIMULATION ALGORITHM AND RESULTS

We simulated a frugal forager for a range of frugality threshold levels in spatial dimensions $d = 1$, $d = 2$, and $d = 3$. In all cases, a direct simulation, in which the full state of the lattice is initially stored in computer memory and continually updated, is prohibitively inefficient. For example, to directly simulate the maximally frugal forager with $\mathcal{S} = 10^4$ in $d = 3$, where the forager lifetime is roughly 10^{10} (see Fig. 4) and therefore a typical displacement is roughly 10^5, would require a lattice of the order of 10^{15} sites.

However, because the forager consumes food only at a sparse subset of sites, it is highly advantageous to only store in computer memory those sites where food has been consumed. For this purpose, we use a binary-search tree to store each such emptied site. The binary tree structure ensures that checking whether a site has been previously visited is fast. The search time scales logarithmically in the number of emptied sites, compared to linearly in the number of emptied sites if these sites were stored in a linear list. Storing only the emptied sites also drastically reduces the memory requirement to scaling linearly with the actual number of emptied sites. Furthermore, not storing the entire lattice eliminates the need to declare, initialize, and update extremely large lattices that are large enough to eliminate finite-size effects. In higher than one dimension, we also use lexicographic ordering of the

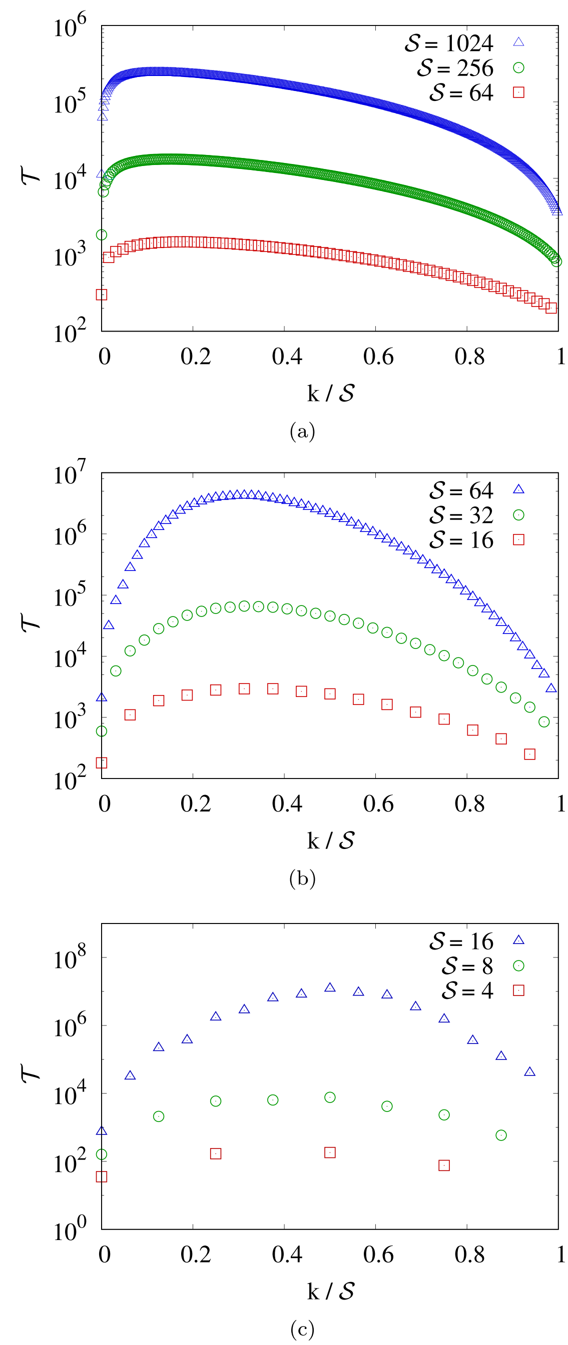

FIG. 2. Simulation results for dependence of the forager lifetime on frugality threshold k for various $\mathcal{S}$ in (a) $d = 1$, (b) $d = 2$, and (c) $d = 3$. The case $k = \mathcal{S} - 1$ corresponds to a normal forager (starving random walk).

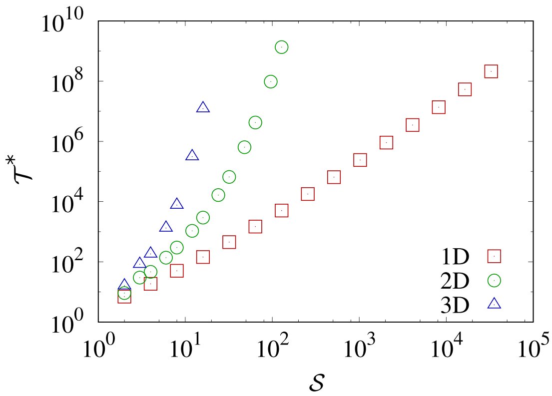

FIG. 3. Simulation results for dependence of the forager lifetime on $\mathcal{S}$ at the optimal frugality threshold k^* in $d = 1$, $d = 2$, and $d = 3$.

coordinates to order the positions of the emptied sites in the binary search tree.

Using this efficient algorithm, we find that the average forager lifetime is maximized at distinct optimal thresholds k^* in $d = 1$, $d = 2$, and $d = 3$ (Fig. 2). Thus, it is advantageous for a forager to tune its frugality level to an optimal value. It is also worth noting that as the value of $\mathcal{S}$ of a forager increases, which correlates with larger body size, the forager maximizes its lifetime by becoming progressively more frugal. That is, k^* is a sublinear function of $\mathcal{S}$.

Finally, we simulated the forager lifetime at the optimal value of the frugality threshold in $d = 1, 2$, and 3. This optimal frugality threshold is determined from plots such as those given in Fig. 2. This maximal lifetime, which we write as $\mathcal{T}^*$, appears to grow as a power law in $\mathcal{S}$ in $d = 1$ (Fig. 3). On the double logarithmic scale of this figure, there is a slight upward curvature in the data for $d = 1$, but the dependence of $\mathcal{T}^*$ on $\mathcal{S}$ appears to be close to quadratic.

For $d = 2$ and $d = 3$, $\mathcal{T}$ clearly grows faster than a power law in $\mathcal{S}$. On a semilogarithmic scale, however, the data for $d = 2$ is curved downward, while last five points of the data for $d = 3$ is nearly linear. A rudimentary analysis suggests that $\mathcal{T} \sim \exp(\sqrt{\mathcal{S}})$ for $d = 2$, while for $d = 3$, the same analysis suggests that $\mathcal{T} \sim \exp(\mathcal{S}^{\nu})$, with ν in the range of 0.8 to 0.9. Thus, it is possible that the dependence is a simple exponential in $d = 3$. The point that we wish to emphasize is that in all dimensions, it is highly advantageous for a forager to tune its frugality threshold to its optimal value.

IV. ANALYTICS

To understand the conditions that optimize the forager lifetime, we first investigate the behavior of a frugal forager in the limit where $k \ll \mathcal{S}$; that is, the forager is extremely frugal. By probabilistic arguments, we will show that the dependence of the lifetime $\mathcal{T}$ on $\mathcal{S}$ becomes progressively more steep for increasing k and that the lifetime also grows more quickly than that of the normal forager (the case $k = \mathcal{S} - 1$). These two facts mandate that there must exist an optimum frugality k^*, where the dependence of the lifetime on $\mathcal{S}$ is the fastest.

A. Maximally frugal forager

We first treat the extreme case of a maximally frugal forager that can only eat if it encounters food at the instant of starvation, i.e., $k = 0$. From investigating this limit, we thereby infer that always consuming resources whenever they are encountered, i.e., the normal forager with $k = \mathcal{S} - 1$, is a suboptimal survival strategy.

To continue to survive, the maximally frugal forager must land on previously unvisited sites at times $\mathcal{S}, 2\mathcal{S}, 3\mathcal{S}$, etc. (For simplicity, we consider even $\mathcal{S}$ and hypercubic lattices.) If the forager lands on a previously visited site (where food was consumed, by definition) at time $m\mathcal{S}$, with m an integer, starvation immediately occurs, and the forager lifetime equals $m\mathcal{S}$. It is worth noting that the maximally frugal forager is equivalent to a self-avoiding flight [35–37], in which each step of the flight is determined by the displacement of a nearest-neighbor random walk of $\mathcal{S}$ steps and in which landing on a previously visited site is not allowed. Thus, the result (6) for the forager survival probability that we will derive below also describes to the survival probability of a self-avoiding flight.

We first show that the survival probability decays exponentially in time in any dimension. Define S_m as the probability that a maximally frugal forager survives until time $m\mathcal{S}$, and let R_m denote the probability for a pure random walk to return to its starting point at time $m\mathcal{S}$. The forager survives its first potential starvation event at time $\mathcal{S}$ with probability $S_1 = 1 - R_1$. We obtain an upper bound for the survival probability at time $2\mathcal{S}$ by demanding that the forager steps to a different site from where it was at time $\mathcal{S}$. The probability for this event is again S_1. Because we have not included the possibility that the forager has returned to the origin at time $2\mathcal{S}$, the true survival probability will be smaller still. Therefore, $S_2 \leqslant (S_1)^2$. Continuing this reasoning gives $S_m \leqslant (S_1)^m$, a result that is valid for any $\mathcal{S}$. Thus, the survival probability is bounded from above by an exponential decay in m.

We obtain a lower bound by noting that the forager is sure to survive if its position always has a positive increment in a single coordinate direction between times $m\mathcal{S}$ and $(m+1)\mathcal{S}$. Let $Q_1(d)$ be the probability that a random walk has a single coordinate equal to zero at time $\mathcal{S}$ in d dimensions. Then the probability that a single coordinate has increased is $\frac{1}{2}[1 - Q_1(d)]$. We therefore have the bounds

$$2d\,\left(\tfrac{1}{2}\right)^m [1 - Q_1(d)]^m \leqslant S_m \leqslant (1 - R_1)^m\,, \qquad (1)$$

so that S_m asymptotically decays exponentially in m and, correspondingly, exponentially in time. However, we will show the mean lifetime of the forager in low spatial dimension is controlled by an intermediate-time regime for large $\mathcal{S}$, where the survival probability decays faster than exponentially in m for $d \leqslant 2$.

We start by deriving an exact recurrence that is satisfied by the survival probability of a maximally frugal forager and then give explicit results for $\mathcal{S} \to \infty$. Formally, the survival probability is given by

$$S_m = \Pr\{\Delta_m = 1, \Delta_{m-1} = 1, \ldots, \Delta_1 = 1\}\,, \qquad (2a)$$

where Δ_m is the indicator function that equals 1 if the forager visits a new site at step $m\mathcal{S}$ and equals 0 otherwise. The above equation merely states that the forager always visits a new site every mth step.

The survival probability satisfies the recursion

$$\begin{aligned} S_m &= \Pr\{\Delta_m = 1|\Delta_{m-1} = 1,\dots,\Delta_1 = 1\} \\ &\quad\times \Pr\{\Delta_{m-1} = 1,\dots,\Delta_1 = 1\} \\ &= \Pr\{\Delta_m = 1|\Delta_{m-1} = 1,\dots,\Delta_1 = 1\}\, S_{m-1}\,. \end{aligned} \tag{2b}$$

To obtain explicit results from this exact recurrence, we make the approximation

$$\Pr\{\Delta_m=1|\Delta_{m-1}=1,\dots,\Delta_1=1\} \simeq \Pr\{\Delta_m=1\}; \tag{3}$$

that is, correlations with past events are ignored. As a parenthetical remark, it is worth noting that our approximation (3) becomes asymptotically exact as $\mathcal{S}\to\infty$. In the "worst" case of $\mathcal{S}=1$, where the maximally frugal forager is equivalent to the self-avoiding walk (SAW), the approximation (3) still provides good results. For $d>2$, our "uncorrelated" approximation gives $\mu_d \simeq 2d(1-R)$ for the SAW connectivity constant μ_d. For the cubic lattice, this formula gives $\mu_3 \simeq 3.96\ldots$, compared with the numerical result $\mu_3 = 4.68\ldots$. The analogous comparisons for higher dimensional hypercubic lattices are $\mu_4 \simeq 6.46\ldots$, compared with $\mu_4 = 6.78\ldots$ (numerical); and $\mu_5 \simeq 8.65\ldots$, compared with $\mu_5 = 8.85\ldots$ (numerical).

Returning to the calculation of the forager survival probability, we need to determine $\Pr\{\Delta_m = 1\}$. For this purpose, it is useful to introduce its generating function, which is known to be [34]

$$\Delta(z) \equiv \sum_{n\geqslant 0}\Pr\{\Delta_n = 1\}z^n = -1 + \frac{1}{(1-z)R(z)}\,. \tag{4}$$

Here $R(z)\equiv\sum_{n\geqslant 0}R_n z^n$ is the generating function for the return probability R_n of a nearest-neighbor random walk at the $(n\mathcal{S})$th step, which has the asymptotic behavior [34]:

$$R_m \simeq 2[d/(2\pi m\mathcal{S})]^{d/2}\,. \tag{5}$$

We now expand $1/R(z)$ as

$$\begin{aligned} &1 - R_1 z + \left(R_1^2 - R_2\right)z^2 - \left(R_1^3 - 2R_1R_2 + R_3\right)z^3 + \cdots \\ &\quad\simeq 1 - \sum_{j\geqslant 1} R_j z^j\,. \end{aligned}$$

In the second line, powers of R_j greater than 1 are neglected compared to linear terms because, for $\mathcal{S}\gg 1$, which we assume throughout, $R_j \sim \mathcal{S}^{-d/2}$.

Substituting the above expansion for $1/R(z)$ in Eq. (4), we obtain the series for $\Delta(z)$, from which we can immediately read off $\Pr\{\Delta_m = 1\} = 1 - \sum_{1\leqslant j\leqslant m} R_j$. Using this expression in Eqs. (2b) and (3) gives the following compact expression for the survival probability:

$$S_m \simeq \exp\left(-\sum_{\ell=1}^{m}\sum_{j=1}^{\ell} R_j\right). \tag{6}$$

Now substituting R_j from (5) in (6), and approximating the double sum by a double integral, we obtain, after several straightforward steps,

$$-\ln S_m \simeq \begin{cases} \sqrt{\frac{32}{9\pi}}\, m^{3/2}\,\mathcal{S}^{-1/2} & d=1\,,\\ \frac{2}{\pi}\, m\ln m\ \mathcal{S}^{-1} & d=2\,,\\ m A_d\ \mathcal{S}^{-d/2} & d>2\,, \end{cases} \tag{7}$$

with $A_d \equiv 2\zeta(d/2)(d/2\pi)^{d/2}$ for $d>2$.

Note that the lower bound (1) for the survival probability imposes the constraint that (7) cannot hold when $m\geqslant\mathcal{S}$ in $d=1$, and when $m\geqslant e^{\mathcal{S}}$ in $d=2$. Imposing this constraint, the average number of "generations" that forager survives in $d=1$ can be found from

$$\langle m\rangle = \sum_{m\geqslant 1} S_m \simeq \int_0^{\beta\mathcal{S}} e^{-\alpha(m^3/\mathcal{S})^{1/2}}\,dm + \int_{\beta\mathcal{S}}^{\infty} e^{-\gamma m}\,dm\,,$$

where $\alpha=\sqrt{32/9\pi}$, and β,γ are constants of order 1 that do not affect the leading asymptotic behavior for large $\mathcal{S}$. It is clear that the integral over the finite range dominates in the above expression for $\langle m\rangle$. Computing this integral and performing similar calculations in higher dimensions leads to

$$\mathcal{T} \simeq \begin{cases} \Gamma(2/3)\left(\frac{\pi}{12}\right)^{1/3}\mathcal{S}^{4/3} & d=1\,,\\ \frac{\pi}{2\ln\mathcal{S}}\ \mathcal{S}^2 & d=2\,,\\ \frac{1}{A_d}\ \mathcal{S}^{1+d/2} & d>2\,. \end{cases} \tag{8}$$

Our numerical results for $d=1,2,3$ agree with the predictions of Eq. (8); see Fig. 4(a).

From (8), the maximally frugal forager in $d=1$ lives longer than the normal forager, whose lifetime is $\mathcal{T}\sim\mathcal{S}$ [15,16]. Moreover, the maximally frugal forager consumes $\mathcal{S}\langle m\rangle \sim \mathcal{S}^{4/3}$ units of food over its lifetime, while the lifetime consumption of the normal forager is $\mathcal{S}^{3/2}$. Despite living a factor of $\mathcal{S}^{1/3}$ longer, the frugal forager asymptotically consumes a factor $\mathcal{S}^{-1/6}$ less resources. Thus extreme frugality in one dimension leads to a longer lifetime and less resource consumption.

B. General frugality threshold

We now generalize to the case of frugality threshold with general $k>0$; here the forager can eat up to k time steps before starvation. For this more general situation, we will show that the average lifetime has an optimum with respect to k.

As a preliminary, we first study the case of $k=1$, where the forager starves when it lands on a doublet of two consecutive empty (previously visited) sites and remains within the doublet at the next step. We exploit the same assumption of lack of correlations (as was used to derive (6) for the maximally frugal forager) to decompose the probability of visiting new sites at times $m\mathcal{S}$ and $m\mathcal{S}+1$ so that we can write

$$\begin{aligned} \Pr\{\Delta_m=1\ \text{ and }\ \Delta'_m=1\} &\simeq \Pr\{\Delta_m=1\}\Pr\{\Delta'_m=1\} \\ &\simeq [\Pr\{\Delta_m=1\}]^2\,. \end{aligned}$$

Here Δ_m is the indicator function to visit a new site at step $m\mathcal{S}$, while Δ'_m is the indicator function to visit a new site at step $m\mathcal{S}+1$. Using this decomposition, the probability to survive

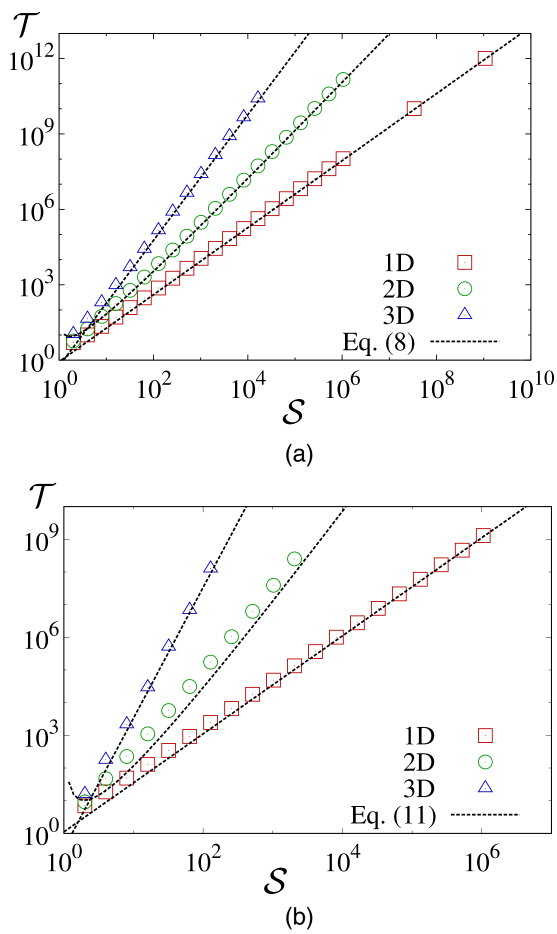

FIG. 4. Simulation results for the lifetime $\mathcal{T}$ of (a) the maximally frugal and (b) the threshold $k = 1$ forager vs $\mathcal{S}$.

m generations is given by

$$S_m(k=1) \simeq \exp\left[-\sum_{\ell=1}^{m}\left(\sum_{j=1}^{\ell} R_j\right)^2\right], \tag{9}$$

where we have again assumed that the states of adjacent sites are uncorrelated with each other. Substituting in the expression for R_m given in (5) and approximating the sums by integrals, we obtain

$$-\ln S_m(k\!=\!1) \simeq \begin{cases} \frac{4m^2}{\pi}\,\mathcal{S}^{-1} & d=1\,, \\ \frac{4m\ln^2 m}{\pi^2}\,\mathcal{S}^{-2} & d=2\,, \\ m A_d^2\,\mathcal{S}^{-d} & d>2\,. \end{cases} \tag{10}$$

Now we follow the steps parallel to those that gave Eq. (8), to find, for the average lifetime $\mathcal{T}$ for $k = 1$,

$$\mathcal{T}(k=1) \simeq \begin{cases} \frac{\pi}{\sqrt{8}}\,\mathcal{S}^{3/2} & d=1\,, \\ \left(\frac{\pi}{4\ln\mathcal{S}}\right)^2\mathcal{S}^3 & d=2\,, \\ \frac{1}{A_d^2}\,\mathcal{S}^{1+d} & d>2\,. \end{cases} \tag{11}$$

These expressions again agree with numerical simulations [Fig. 4(b)]. It is important to note that the lifetime for the $k = 1$ forager exceeds that of the maximally frugal forager ($k = 0$). Because the lifetime of the normal forager ($k = \mathcal{S} - 1$) is shorter than that of the maximally frugal forager, there must be an intermediate frugality value k^* that maximizes the lifetime.

TABLE I. Comparison between simulation results for the exponent τ in $\mathcal{T} \sim \mathcal{S}^\tau$ in one dimension (top row) and our analytical predictions (bottom row): Eq. (8) for $k = 0$, Eq. (11) for $k = 1$, and then the first line of (14) for $k > 1$.

	$k=0$	1	2	4	8	16	32	64	128
Simul.	1.33	1.53	1.53	1.56	1.63	1.69	1.76	1.79	1.83
Analytic	1.33	1.50	1.53	1.61	1.69	1.76	1.82	1.86	1.90

We now extend the calculational approach for $k = 1$ to general frugality threshold k, with $k \ll \mathcal{S}$. For a forager to starve in one dimension, it first has to be metabolically depleted to its frugality threshold, then step to the interior of a gap of consecutive previously visited sites, and finally make k subsequent steps within this gap. The average length of a gap that will trap the forager is simply $\langle N(k)\rangle$, the mean number of distinct sites visited by a random walk of k steps [33,34]. Following the same reasoning as that applied for the case $k = 1$, we obtain

$$S_m(k) \simeq \exp\left[-\sum_{\ell=1}^{m}\left(\sum_{j=1}^{\ell} R_j\right)^{\langle N(k)\rangle}\right]. \tag{12}$$

Here we make the uncontrolled approximation that the survival probability averaged over all random-walk trajectories can be obtained by averaging the number of distinct sites visited in the exponent of the above expression.

Substituting the return probability (5) for nearest-neighbor random walks into (12), the leading behavior of the survival probability is

$$-\ln S_m(k) \sim \begin{cases} m^{1+\langle N(k)\rangle/2}/\mathcal{S}^{\langle N(k)\rangle/2} & d=1\,, \\ m(\ln m)^{\langle N(k)\rangle}/\mathcal{S}^{\langle N(k)\rangle} & d=2\,, \\ m/\mathcal{S}^{d\langle N(k)\rangle/2} & d>2\,. \end{cases} \tag{13}$$

From these results, we obtain the lifetime

$$\mathcal{T}(k) \sim \begin{cases} \mathcal{S}^{(2\langle N(k)\rangle+2)/(\langle N(k)\rangle+2)} & d=1\,, \\ \mathcal{S}^{\langle N(k)\rangle+1}/(\ln\mathcal{S})^{\langle N(k)\rangle} & d=2\,, \\ \mathcal{S}^{1+d\langle N(k)\rangle/2} & d>2\,, \end{cases} \tag{14}$$

with the asymptotic behavior of $\langle N(k)\rangle$ for $k \gg 1$ given by [33,34] $\sqrt{8k/\pi}$ ($d = 1$), $\pi k/\ln k$ ($d = 2$), and $k/R(1)$ ($d = 3$).

Because of the uncontrolled nature of the approximation in (12), one should not anticipate that our prediction for the dependence of $\mathcal{T}$ on $\mathcal{S}$ for different frugality thresholds k will match simulation results quantitatively. However, these two results are gratifyingly close in spite of the crudeness of our approach (Table I).

V. SUMMARY

We introduced a family of foraging models that are characterized by two parameters—the amount of food per site $\mathcal{S}$ (measured in units of the amount metabolized by the forager in a single step) and the frugality threshold k, the number of steps before starvation when the forager can consume food. The frugal forager model is simple to formulate but is technically challenging, as the dynamics is both stochastic and inherently non-Markovian. The simplest version of our model, namely, the case $k = 0$ and $\mathcal{S} = 1$, reduces to the standard self-avoiding walk, which is well understood only in low spatial dimensions. For our frugal forager model, the spatial dimension plays a crucial role, and the attribute of frugality introduces a new layer of complexity into the dynamics. The interplay between *conservation* (not eating) and *consumption* leads to a rich dynamics in which the lifetime of the forager is maximized at an optimal level of frugality. While it naively seems that being frugal is inherently risky, this strategy turns out to be superior to that of the normal forager, which always eats when it encounters food. We also extended our approach to obtain the forager lifetime in any dimension and for general frugality threshold $k \ll \mathcal{S}$ by exploiting the classic formalism for visits to distinct sites of a random walk.

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