Reply to Comment by Singh and Rajesh on "Growth Inside a Corner: The Limiting Interface Shape"

Jason Olejarz,[1] P. L. Krapivsky,[2] S. Redner,[1] and K. Mallick[3]
[1] *Center for Polymer Studies, and Department of Physics, Boston University, Boston, MA 02215, USA*
[2] *Department of Physics, Boston University, Boston, MA 02215, USA*
[3] *Institut de Physique Théorique CEA, IPhT, F-91191 Gif-sur-Yvette, France*

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In Ref. [1], we investigated a two-dimensional interface that grows in one octant of the cubic lattice. Exploiting limiting cases and symmetry, we conjectured two basic nonlinear equations of motion for the interface speed. Combining these equations allows us to fit the interface speed along the $(1,1,1)$ diagonal perfectly, but this fitting requires an unnatural choice of fitting parameters. Aesthetics suggests that one of our elemental equations,

$$z_t = \frac{z_x}{z_x-1}\,\frac{z_y}{z_y-1}\left[1-\frac{1}{z_x+z_y}\right] \equiv R\,, \qquad (1)$$

accurately describes corner $3d$ interface growth. While the prediction from (1), $w = 0.125$, for the interface speed in the $(1,1,1)$ direction accurately matches our measured value $w = 0.1261(2)$, small discrepancies persist. The comment [2] studies the same interface growth rules starting from a flat interface perpendicular to $(1,1,1)$. This work finds $w = 0.12606(2)$, consistent with our numerics, but also in slight disagreement with the solution to the conjectured exact equation.

We recently found other independent equations that satisfy the required symmetries. One example is

$$z_t = R \times \frac{(1-z_x-z_y)^n}{1+(-z_x)^n+(-z_y)^n} \qquad (2)$$

for arbitrary n. Setting $n = 1+\log_3(8w)$ perfectly matches numerics; for $w = 0.12606$, $n = 1.0077$. However, the conjecture (1) is aesthetically more compelling.

It seems coincidental that the beautifully symmetric growth equation (1) should differ from simulations by less than a percent. It is also unsatisfying to reproduce the numerics with high accuracy by using an equation such as (10) in our original Letter or (2), which contain unnatural fitting parameters. We speculate that systematic effects in the simulations may generate small discrepancies with the prediction of (1). Also note that little is known analytically about long-lived transients in $(2+1)$-dimensional Kardar-Parisi-Zhang growth [3]. A recent numerical study [4] reveals a similarly stubborn approach to asymptotics in measurements of scaling exponents for KPZ growth models. On a similar note, it is conceivable that differences in height correlation functions between the flat hypercube-stacking model [5] of the comment and the curved corner interface that we examine may generate slight differences in the interface speed w. Such differences in interfacial statistics between flat and curved geometries have been found rigorously in analogous $(1+1)$-dimensional growth models [6].

While we agree with [2] that the numerics slightly deviate from the predictions of (1), it seems rash to reject this simple equation of motion in favor of an unnaturally complex one that minutely improves the accuracy for the interface speed. The outstanding challenge, of course, is to derive the correct equation for the interface motion.

[1] J. Olejarz, P. L. Krapivsky, S. Redner, and K. Mallick, Phys. Rev. Lett. **108**, 016102 (2012).
[2] R. Singh and R. Rajesh, preceding Comment.
[3] For a review, see I. Corwin, Random Matrices: Theory Appl. **01**, 1130001 (2012).
[4] T. Halpin-Healy, Phys. Rev. Lett. **109**, 170602 (2012).
[5] L.-H. Tang, B. M. Forrest, and D. E. Wolf, Phys. Rev. A **45**, 7162 (1992).
[6] M. Prähofer and H. Spohn, Phys. Rev. Lett. **84**, 4882 (2000).