Statistical Mechanics of Popularity

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Motivations:

- Traffic clustering on "Suicide Alley"
- Internet mapping
- Citation distribution
- City population distribution

Dimensional Analysis for Traffic Clustering

Empirical Analysis of Citation Data

Models of Growing Networks

Rate Equation for Citations and the Internet

Rate Equation for City Populations

Outlook

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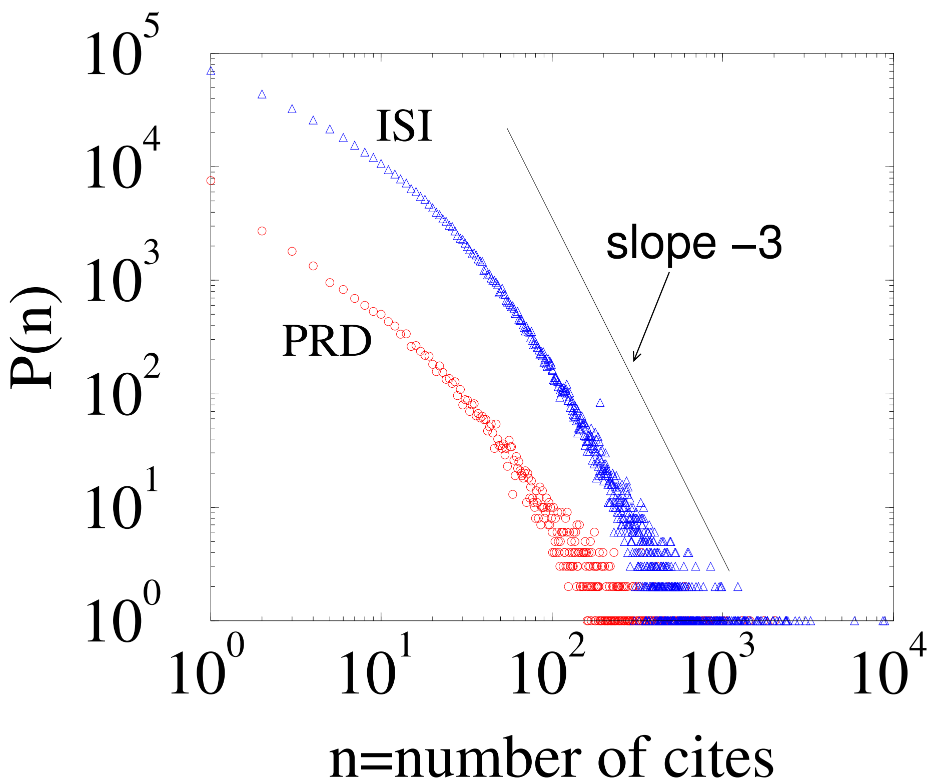
Paul Krapivsky (Boston University)
Francois Leyvraz (CIC, Mexico)
Geoff Rodgers (Brunel University, UK)

Citation Distribution

ISI: 783,339 papers, 6,716,198 cites, $\langle n \rangle = 8.57$.

1	paper	cited	8907	times
64	papers		>1000	times
282	papers		>500	times
2103	papers		>200	times
633391	papers		<10	times
368110	papers		0	times!

PRD: 24,296 papers, 351,872 cites, $\langle n \rangle = 14.48$,



Analysis of Citation Data

If the citation distribution has the form

$$P(n) \sim n^{-\mu} \qquad \text{with} \quad \mu > 2,$$

then the number of cites of the most popular paper is determined by

$$\int_{n_{\max}}^{\infty} n^{-\mu}\, dn = \frac{1}{N}.$$

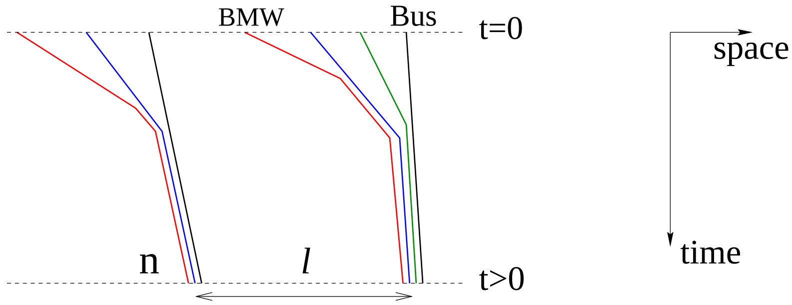
This gives $n_{\max} \sim N^{1/(\mu-1)}$.

k^{th} most popular paper has $\left(\frac{N}{k}\right)^{1/(\mu-1)}$ cites.

Zipf plot: # of cites of k^{th} ranked paper vs. k:

$$n(k) \sim k^{-1/(\mu-1)}.$$

Space-Time Traffic Evolution



Dimensional analysis: BKR (1994)

$$Q_+(v) = \int_v^\infty P_0(v')\,dv' \qquad \text{fast car probability}$$

$$Q_-(v) = 1 - Q_+(v) \qquad \text{slow car probability}$$

Then

$$n(v) \approx \sum_{k=1}^{\infty} k\, Q_+^k\, Q_- = Q_+/Q_-$$

$$\sim v^{-(1+\mu)} \qquad \text{for } P_0(v) \sim v^\mu$$

This gives: $n(v) \sim v^{-(1+\mu)} \sim \ell \sim vt$

Basic result:

$$n \sim t^{\mu+1/\mu+2} \qquad v \sim t^{-1/(\mu+2)}$$

City Population Dynamics

Migration/Growth Model

Migration:

$$c_i + c_j \xrightarrow{K(k,l)} c_{i+1} + c_{j-1} \qquad i > j$$

with $K(ai, aj) \sim a^\lambda K(i, j)$.

Demographic growth:

$$c_j \xrightarrow{j\gamma} c_{j+1}$$

Basic Fact:

small cities shrink, big cities grow