# Pick up handout at front of hall 

10.45 start


## Emergence

Automata Decomposition, Innovation, and Dissipation

Lecture 4
10:45 AM
28 June 2011


Effective Theories for Circuits and Automata http://arxiv.org/abs/I I06.5778

## Topics

## Hierarchies of Computation

The Chomsky Hierarchy
Into the Labyrinth
Finite State Automata
Non-deterministic vs. Probablistic

## Groups

Review of Group and Semigroup Theory
Jordan-Holder Decomposition
Semigroups and Automata
The Amazing Krohn-Rhodes Theorem
How to "Smooth" a Computer
Universal vs. Finite State Computation Introducing Stochasticity

Prime Decomposition of the Bengalese Finch

## The Chomsky Hierarchy

## Level 3

Regular

## Level 2

Context Free

Level I
Context Sensitive

Level 0

## "Unrestricted"

## Jim's Bestiary



Figure 2 The discrete computation hierarchy. Adjective legend: $1=$ one way input tape, $2=$ two way input tape, $\mathrm{D}=$ deterministic, $\mathrm{N}=$ nondeterministic, $\mathrm{I}=$ indexed, $\mathrm{RI}=$ restricted $\mathrm{I}, \mathrm{n}=$ nested, $\mathrm{NE}=$ nonerasing, $\mathrm{CF}=$ context free, $\mathrm{CS}=$ context sensitive, $\mathrm{R}=$ recursive, $\mathrm{RE}=\mathrm{R}$ enumerable, and $\mathrm{U}=$ universal. Object legend: $\mathrm{G}=$ grammar, $\mathrm{A}=$ automata, $\mathrm{FA}=$ finite A , PDA $=$ pushdown $\mathrm{A}, \mathrm{SA}=$ stack $\mathrm{A}, \mathrm{LBA}=$ linear bounded $\mathrm{A}, \mathrm{RPA}=$ Reading PDA, TM $=$ Turing machine, $\mathrm{LS}=$ Lindenmayer system, $0 \mathrm{~L}=\mathrm{CF} \mathrm{LS}, 1 \mathrm{~L}=\mathrm{CS} \mathrm{LS}$, and $\mathrm{RS}=\mathrm{R}$ set. (After [31,35-39].)

## Finite State Automata



## Finite State Automata



## Finite State Automata



Figure 2: Graphical representation of the TCP FSM. The events are the flag combinations of Table 4 and the states are as defined in Table 3. For clarity, the failure state is not shown.


#### Abstract

Finite state machines can be used to detect anomalous behaviour in TCP traffic by describing the progression of a connection through states as a result of events based on header flags. The method was applied to real traffic to understand its realistic use and it was found that for the time period analysed here, on the order of $37 \%$ of TCP connections do not follow the TCP protocol specifications. The majority of these are a result of malicious activity, and approximately $4 \%$ are due to benign anomalies such as unresponsive hosts and misconfigurations. The method may be applied as a network security measure, as a network management tool or as a research tool for the study of TCP behaviour on the Internet.


## Finite State Automata



## Hierarchical Decomposition



## Factoring the Z4 Machine



FIG. 3. A cascade of two $Z_{2}$ counters; the lower-level machine has two possible transitions, depending on the state of the higher-level machine. The cascade now can count modulo four, with the high-level machine counting even versus odd, and the lower-level machine tracking the second bit.

## Finite State Automata



## Krohn-Rhodes : all FSA are

ATLAS
FINITE GROUPS

| J. | H. | CO N WA Y |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| R. | T. | C U R T I | S |
| S. | P. | N O R T ON |  |
| R. | A. P A R K E R |  |  |
| R. | A. W I LS ON |  |  |

CLARENDONPRESS:OXFORD

## + "Pure" Resets



## An Example of the Krohn-Rhodes Holonomy Decomposition



## Another Example



FIG. 4. Effective theories and "smoothed" computations. (a) An eight-state automaton with two input letters, built from XOR gates on the left-shift register. (b) The top level of the decomposed machine. Despite the complicated internal structure of (a), at the coarse-grained level, the process is seen count modulo three (on receiving $a$ signals), and reset (on receiving $b$ signals.) Movement within the three superstates of (b) is dictated by $Z_{2}$ counters and resets at lower levels in the decomposition.

## Stochasticity — Random Transitions



## Stochasticity — Random Transitions



FIG. 5. Decomposition of a (single input letter) probabilistic automaton into a 4 -letter deterministic machine; $\pi(a)=$ $\pi(c)=3 / 8 ; \pi(b)=\pi(d)=1 / 8$.

## Ashes to Ashes



FIG. 6. The rise, and fall, of group structures in noisy Boolean systems. For the XOR function of Fig. 2b, with a $1 \%$ bit-flip rate, we show the probability that the function implements one of the group elements of the noise free case ( $Z_{3}$, solid line), one of the four reset automata (dotted line), or any group element not in $Z_{3}$ (the set $S_{4}-Z_{3}$, dashed line.) The light gray line shows the remainder. As time passes and errors accumulate, the chances of a transformation being a member of $Z_{3}$ decline, while the reset automata come to dominate the long term input-output maps. The non-trivial relationship between noise and complexity can be seen when, for a brief period, representatives of more complex groups appear and even dominate the pure resets.

## The Prime Decomposition of the Bengalese Finch


[Finch movie]

# Complex sequencing rules of birdsong can be explained by simple hidden Markov processes Kentaro Katahira, Kenta Suzuki, Kazuo Okanoya, Masato Okada (http://arxiv.org/abs/IOII.2575) 



## The Prime Decomposition of the Bengalese Finch



10 states $\rightarrow 10^{10}$ elements in the full transformation semigroup?

