Calibrating the deadweight loss and factor shares

Here we construct a very simple model that can be calibrated to estimate the magnitude of the overall efficiency cost, or *deadweight loss*, due to labor monopsony, along with its implications for the labor and capital shares. We begin with the most general version of the model, and in subsequent sections investigate simpler settings as limiting cases.

1 Main model

1.1 Setup

Production: We consider a unit mass of many firms producing according to an identical technology, which we take to be a constant elasticity of substitution (CES) aggregation of labor and capital inputs:

$$Y = f(K,L) = A\left[\alpha K^{\frac{\sigma-1}{\sigma}} + (1-\alpha)L^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

Here σ is the elasticity of substitution between capital and labor. This nests a Cobb-Douglas production function when $\sigma = 1$ (Sections 2 and 4), Leontieff production when $\sigma = 0$, and perfect substitution when $\sigma \to \infty$. We normalize $p = A = 1$ where p is the price level, so that all quantities are measured in dollars relative to the current price level.

Labor supply: We model labor supply as taking a two-stage nested structure, in which agents decide whether to work based on the prevailing wage rate, and then make a choice of employer conditional on working. Each firm j faces an isoelastic labor supply curve $L(w_j) = \bar{L}^{firm}(w_{-j})w_j^{\beta}$, where $\beta > 0$ is the elasticity of residual labor supply, w_j is the wage offered by firm j, and w_{-j} is the vector of wages offered by other firms. Since firms are identical, we seek symmetric a equilibrium in which they find it optimal to to post the same wage $w_j = w$ and hire the same amount of labor $L = \bar{L}^{firm}(w)w^{\beta}$ where $\bar{L}^{firm}(w) = \bar{L}^{firm}(w, w, \ldots)$. Facing a wage of w conditional on employment, workers supply labor according to a separate participation elasticity $\eta > 0$, with total employment in the economy $L^{working} = \bar{L}w^{\eta}$ and $\bar{L}$ a parameter.[1] As we've normalized the mass of firms to one, we must have $L^{working} = L$ and hence $\bar{L}^{firm}(w) = \bar{L}w^{\eta-\beta}$. However, we emphasize that as we assume firms are small enough compared with the overall labor market that they ignore the effect of their wage setting on aggregate labor supply, they treat $\bar{L}^{firm}(w)$ as fixed in choosing w_j.

Government: To add a bit more realism, we allow for for an existing wedge between wages earned and the marginal product of labor, in the form of a tax on labor that is used to fund public goods. Thus, imperfect competition for labor will not be assumed

[1] This nests a perfectly competitive model of the labor market as $\beta \to \infty$. In this limit, firms have no discretion over wages and labor supply is determined only along the participation margin according to the labor/leisure tradeoff parameterized by η.

to be the only distortion in the economy, but we consider the case without government in Sections 2 and 3. We continue to let w denote the price paid per unit of labor by the firm, but now suppose that only $(1-\tau)w$ goes to the worker and the remaining τw is revenue to the government. This revenue is then paid back to workers with a public goods value multiplier of m, i.e. output from the government is $G := m\tau w L$.

Our primary quantity of interest will be the the relative deadweight loss that can be attributed to labor market monopsony, that is: how much lower is total output $Y + G$ compared with a world in which $\beta \to \infty$? To this end, we consider Y and G each as functions of β and define $Y^* = \lim_{\beta\to\infty} Y(\beta)$ and $G^* = \lim_{\beta\to\infty} G(\beta)$,the deadweight loss due to monopsony is defined as:

$$DWL := \frac{Y^* + G^* - Y(\beta) - G(\beta)}{Y^* + G^*}$$

When the labor tax and government provision of public goods is taken out of the model $\tau = 0$ (as we'll consider in Sections 2 and 3), DWL is just the percent loss in output Y:

$$DWL := \frac{Y^* - Y(\beta)}{Y^*}$$

1.2 Solving the model

To evaluate the quantity DWL, we begin with the assumption that our representative firm maximizes profits, given the residual labor supply curve $L(w)$ it faces. We take capital to be supplied perfectly elastically at an exogenous rate r. Thu, firms choose w and K to maximize the function

$$\begin{aligned}\Pi(K,w) &= f(K, L(w)) - wL(w) - rK \\ &= \left[\alpha K^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\left\{\bar{L}^{firm}\cdot(1-\tau)^{\beta}w^{\beta}\right\}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} - \bar{L}^{firm}\cdot(1-\tau)^{\beta}w^{\beta+1} - rK\end{aligned}$$

where we suppress the dependence of $\bar{L}^{firm}$ on the prevailing wage rate since each firm treats it as fixed. Assuming interior solutions, this yields capital and labor shares and hence the capital/labor ratio as a function of parameters and the wage. In particular, the first order condition for the wage (and hence labor $L = L(w)$) is

$$\left(\frac{Y}{L}\right)^{\frac{1}{\sigma}} = \frac{\beta+1}{\beta(1-\alpha)}w \tag{1}$$

and for capital

$$\left(\frac{Y}{K}\right)^{\frac{1}{\sigma}} = \frac{r}{\alpha}$$

Thus $K/L = \left(\frac{\alpha}{1-\alpha}\frac{\beta+1}{\beta}\frac{w}{r}\right)^{\sigma}$. Given the CES functional form, this lets us express production Y as a linear function of labor, with a coefficient that depends on this capital labor ratio, i.e

$$Y = L\left[1-\alpha+\alpha\left(\frac{K}{L}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} = L\left[1-\alpha+\alpha\left(\frac{\alpha}{1-\alpha}\frac{\beta+1}{\beta}\frac{w}{r}\right)^{\sigma-1}\right]^{\frac{\sigma}{\sigma-1}}.$$

Substituting back into Equation (1) we we see that the wage is pinned down from the firm's problem as a function of β, α, r and σ. Solving for $\tilde{w}$ yields:[2]

$$\tilde{w} = \left(\frac{(1-\alpha)^{\sigma}}{1-\alpha^{\sigma}r^{1-\sigma}}\right)^{\frac{1}{\sigma-1}} \tag{2}$$

where we define $\tilde{w} := \frac{\beta+1}{\beta}w$. It can be seen from Equation (1) that the quantity $\tilde{w}$ is in fact the marginal product of labor, where $\frac{\beta}{\beta+1} \leq 1$ represents the markdown of wages below labor's marginal product. The "markdown-adjusted" wage $\tilde{w}$ is a useful quantity for our analysis because its value does not depend on the presence of strength of monopsony power, as β does not appear on the RHS of Equation (2).

Now that the value of the wage is known in terms of parameters, we use the participation labor function to pin down the overall labor level and compute production Y. In particular, $L = \bar{L}(1-\tau)^{\eta}w^{\eta}$, so output is, as a function of β and $\tilde{w}$:

$$Y(\beta) = (1-\tau)^{\eta}\bar{L}\left(\frac{\beta}{\beta+1}\right)^{\eta}\left[(1-\alpha)^{\sigma}+\alpha^{\sigma}r^{1-\sigma}\tilde{w}^{\sigma-1}\right]^{\frac{\sigma}{\sigma-1}}\tilde{w}^{\eta} \tag{3}$$

Public goods provision is also linear in labor, and can be written using Equation 1 as

$$G(\beta) = \left(m\tau\frac{\beta}{\beta+1}(1-\alpha)^{\sigma}\tilde{w}^{1-\sigma}\right)Y(\beta) \tag{4}$$

To ease notation, we introduce $\gamma := m\tau(1-\alpha)^{\sigma}\tilde{w}^{1-\sigma}$, which like $\tilde{w}$ is a quantity that does not depend on β.

With Y^* and G^* denoting the values that would occur without monopsony (as defined in Section 1.1), we have from Equation 3 that $Y(\beta) = \left(\frac{\beta}{\beta+1}\right)^{\eta}Y^*$ and hence

$$Y(\beta)+G(\beta) = \left(1+\frac{\beta}{\beta+1}\gamma\right)\left(\frac{\beta}{\beta+1}\right)^{\eta}Y^* = \frac{1+\frac{\beta}{\beta+1}\gamma}{1+\gamma}\left(\frac{\beta}{\beta+1}\right)^{\eta}(Y^*+G^*)$$

Thus, the reduction in total output due to monopsony power is:

$$DWL = \frac{Y^*+G^*-Y(\beta)-G(\beta)}{Y^*+G^*} = 1-\frac{1+\frac{\beta}{\beta+1}\gamma}{1+\gamma}\left(\frac{\beta}{\beta+1}\right)^{\eta} \tag{5}$$

Since public goods all flow to labor, the total share of output to labor will be

$$\frac{wL+G}{Y+G} = \frac{\beta}{\beta+1}\cdot\frac{(1-\alpha)^{\sigma}\tilde{w}^{1-\sigma}+\gamma}{1+\frac{\beta}{\beta+1}\gamma}$$

where we've used Equation 1 to express wL in terms of Y. Similarly, using the first order condition for capital we have:

$$\frac{rK}{Y+G} = \frac{\alpha^{\sigma}r^{1-\sigma}}{1+\frac{\beta}{\beta+1}\gamma}$$

[2]The limit of this expression in the case of Cobb-Douglas production $\sigma = 1$ is not obvious, but in Section 2 we show $\tilde{w}$ to be $(1-\alpha)\left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}}$ when $\sigma = 1$.

and to firm profit:

$$\frac{\Pi}{Y+G} = \frac{Y+G-(wL+G)-rK}{Y+G} = \frac{1-\frac{1}{\beta+1}\gamma - \frac{\beta}{\beta+1}(1-\alpha)^\sigma \tilde{w}^{1-\sigma} - \alpha^\sigma r^{1-\sigma}}{1+\frac{\beta}{\beta+1}\gamma}$$

Another quantity of interest is the *fiscal loss*, or relative loss in tax revenue due to monopsony. Since tax revenue is G/m, this is simply (using Equation 4):

$$\frac{G^* - G(\beta)}{G^*} = 1 - \frac{\beta}{\beta+1}\frac{Y(\beta)}{Y^*} = 1 - \left(\frac{\beta}{\beta+1}\right)^{\eta+1}$$

A final quantity of interest is the relative loss in employment due to monopsony, which works out to be

$$\frac{L(\beta) - L^*}{L^*} = 1 - \left(\frac{\beta}{\beta+1}\right)^{\eta}$$

1.3 Calibration

We calibrate this model with the values $m = 1.3$, $\tau = 0.3$, $\alpha = 1/3$, and a return on capital of $r = 0.04$. Raval (2015) reports a range of estimates for σ between 0.3 and 0.5, and we take $\sigma = 0.4$ as a central figure (in Figure 5 of Section 3 we show factor shares as a function of σ, with $\tau = 0$ and a fixed β). These parameter values yield $\gamma = 0.26$: public goods provision amounts to 26% of private production. Chetty et al. (2014) provides two possible values of the overall labor participation elasticity η: the "aggregate hours elasticity" of 0.5 and the "aggregate extensive-margin" elasticity of 0.17. We take $\eta = 0.3$ as a central figure.

In Table 1 we report results with these parameter values and a variety of estimates for the residual labor supply elasticity β from the literature: Staiger et al. (2010) (registered nurses), Dube et al. (2017a) (online labor markets), Ransom and Sims (2010) (schoolteachers) as well as Isen (2013), Dube et al. (2017b) (taking a central estimate of $\eta = 3$) and Kline et al. (2017), all for broad populations of US workers. We also impute elasticities from the Herfindahl index HHI of labor market concentration reported in Azar et al. (2017), using the relationship $\beta = \frac{\eta}{HHI/10000}$ that holds under a Cournot model of competition. We use their mean estimate of the HHI across job vacancies $HHI = 3157$, which with $\eta = 0.3$ yields $\beta = 0.95$. Figure 1 plots DWL and factor shares as a continuous function of β. In Figure 2, we plot the percentage deadweight loss alongside the percentage reductions in employment L, and revenue to the government G/m.

2 Cobb-Douglas production, no government

A much simpler version of the model specializes that of Section 1 to a Cobb-Douglas production technology $\sigma = 1$ and no government distortion: $\tau = 0$. For clarity, we work this case out separately, although it can be obtained by taking appropriate limits from our

Table 1: CES ($\sigma = 0.4$) estimates under a $\tau = 30\%$ labor tax with efficiency multiplier of $m = 1.3$.

β	Implied markdown	β source	**DWL**	Labor share	Profit share	Fiscal loss
0.1	0.09	Staiger et al. (2010)	**62.9%**	11%	80%	96%
0.12	0.11	Dube et al. (2017a)	**60.8%**	13%	78%	95%
0.95	0.49	Azar et al. (2017)	**30.2%**	52%	40%	61%
2	0.67		**19.2%**	68%	24%	41%
2.7	0.73	Kline et al. (2017)	**15.4%**	73%	19%	34%
3	0.75	Dube et al. (2017b)	**14.3%**	75%	18%	31%
3.7	0.79	Ransom and Sims (2010)	**12.1%**	78%	15%	27%
5.5	0.85	Isen (2013)	**8.7%**	82%	11%	20%
10	0.91		**5.1%**	87%	6%	12%
100	0.99		**0.6%**	92%	1%	1%

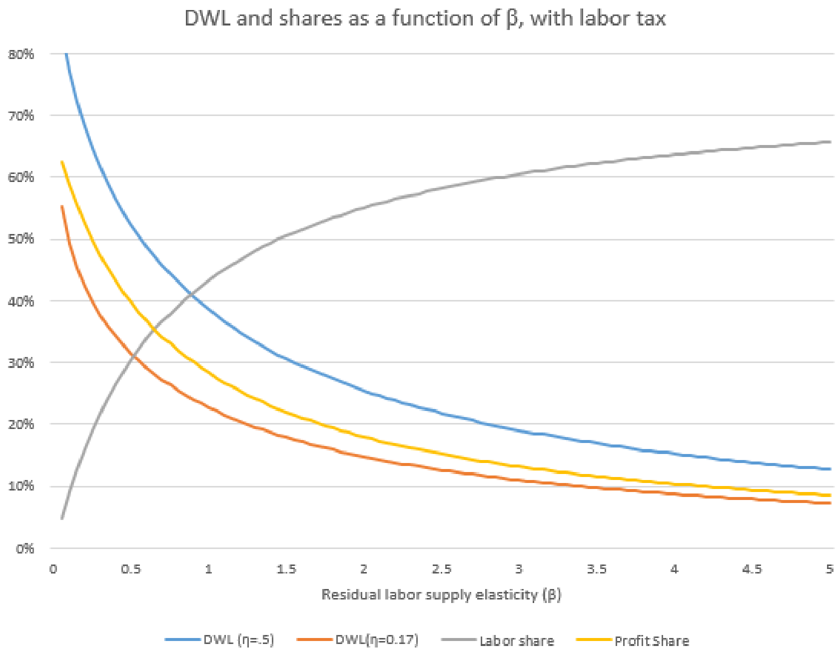

Figure 1: Deadweight loss and labor/profit shares under a $\tau = 30\%$ labor tax with efficiency multiplier of $m = 1.3$, CES production with $\sigma = 0.4$

results in Section 1 (these limits are not trivial to work out). Consider a Cobb-Douglas, constant-returns to scale production function

$$Y = f(K, L) = AK^{\alpha}L^{1-\alpha}$$

where $\alpha \in [0, 1]$ and we again normalize $p = A = 1$. This output is allocated to the owners of capital, workers, and profits as:

$$Y = rK + wL + \Pi. \tag{6}$$

Let w^* be the competitive wage that would prevail if there were no monopsony power in the labor market. Then:

$$Y^* = K^{*\alpha}L^{*1-\alpha} = rK^* + w^*L^*$$

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Figure 2: Deadweight loss and percentage changes in employment and government revenues under a $\tau = 30\%$ labor tax with efficiency multiplier of $m = 1.3$, CES production with $\sigma = 0.4$

where K^* and L^* are the aggregate levels of capital and labor that would occur in this no-monopsony counterfactual, and the second equality follows analogously to Equation (6) but with no profits. As before, we're interested in the relative deadweight-loss due to monopsony: $DWL = \frac{Y^* - Y(\beta)}{Y^*}$.

We begin by solving for Y^*. By profit maximization, we get the standard Cobb-Douglas factor share equations: $\frac{rK^*}{Y^*} = \alpha$ and $\frac{w^* L^*}{Y^*} = 1 - \alpha$, and dividing the two we have that $\frac{K^*}{L^*} = \frac{\alpha}{1-\alpha} \frac{w^*}{r}$. Then from the labor equation we have

$$w^* = MRPL = (1-\alpha)\frac{Y^*}{L^*} = (1-\alpha)\left(\frac{K^*}{L^*}\right)^{\alpha} = (1-\alpha)^{1-\alpha}\left(\frac{\alpha}{r}\right)^{\alpha} w^{*\alpha}$$

and we can solve for the wage: $w^* = (1-\alpha)\left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}}$. To close the model, we use the aggregate participation labor supply function $L = \bar{L} w^{\eta}$. Labor market clearing then implies that $L^* = \bar{L}(1-\alpha)^{\eta}\left(\frac{\alpha}{r}\right)^{\frac{\eta\alpha}{1-\alpha}}$, and aggregate output is

$$Y^* = \frac{w^* L^*}{1-\alpha} = \bar{L}(1-\alpha)^{\eta}\left(\frac{\alpha}{r}\right)^{\frac{(\eta+1)\alpha}{1-\alpha}}$$

Now we consider the monopsony model. Recall that employers are homogeneous and face a residual labor supply curve $L(w) = \bar{L}^{firm} w^{\beta}$, where $\beta > 0$. The first order conditions for profit maximization now imply that the factor share paid to labor is $\frac{wL}{Y} = (1-\alpha)\frac{\beta}{\beta+1}$ while the factor share to capital remains unchanged at $\frac{rK}{Y} = \alpha$. Profit as a share of output is thus $\frac{\Pi}{Y} = \frac{Y-wL-rK}{Y} = \frac{1-\alpha}{\beta+1}$. All firms offer the same wage, which satisfies

$$w = \frac{\beta}{\beta+1} MRPL = \frac{\beta}{\beta+1}(1-\alpha)\left(\frac{K}{L}\right)^{\alpha} = \left(\frac{\beta}{\beta+1}(1-\alpha)\right)^{1-\alpha}\left(\frac{\alpha}{r}\right)^{\alpha} w^{\alpha}$$

Solving for the wage, as before: $w = \frac{\beta}{\beta+1}(1-\alpha)\left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}}$, a simple markdown of $\frac{\beta}{\beta+1}$ from w^*,[3] and we again let aggregate labor participation be given by $L(w) = \bar{L}w^{\eta}$. We now use the relation $Y = wL\frac{\beta+1}{\beta(1-\alpha)} = \frac{\bar{L}(\beta+1)}{\beta(1-\alpha)}w^{\eta+1}$ to get:

$$\begin{aligned} Y &= \frac{\bar{L}(\beta+1)}{\beta(1-\alpha)}\left(\frac{\beta}{\beta+1}(1-\alpha)\right)^{\eta+1}\left(\frac{\alpha}{r}\right)^{\frac{(\eta+1)\alpha}{1-\alpha}} \\ &= \bar{L}\left(\frac{\beta}{\beta+1}(1-\alpha)\right)^{\eta}\left(\frac{\alpha}{r}\right)^{\frac{(\eta+1)\alpha}{1-\alpha}} = Y^*\left(\frac{\beta}{\beta+1}\right)^{\eta} \end{aligned}$$

The relative deadweight loss is thus: $DWL = 1 - \left(\frac{\beta}{\beta+1}\right)^{\eta}$. Table 2 and Figure 3 report numerical results for this case with the calibration from Section 1.3.

Table 2: Estimates with participation elasticities η from Chetty et al. (2014) and $\alpha = 1/3$

β	Implied markdown	β source	**DWL**	Labor share	Profit share
0.1	0.09	Staiger et al. (2010)	**51.3%**	6%	61%
0.12	0.11	Dube et al. (2017a)	**48.8%**	7%	60%
0.95	0.49	Azar et al. (2017)	**19.4%**	32%	34%
2	0.67		**11.5%**	44%	22%
2.7	0.73	Kline et al. (2017)	**9.0%**	49%	18%
3	0.75	Dube et al. (2017b)	**8.3%**	50%	17%
3.7	0.79	Ransom and Sims (2010)	**6.9%**	52%	14%
5.5	0.85	Isen (2013)	**4.9%**	56%	10%
10	0.91		**2.8%**	61%	6%
100	0.99		**0.3%**	66%	1%

[3] In the notation of Section 1.2, we have thus that $\tilde{w} = (1-\alpha)\left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}}$ in the $\sigma = 1$ limit.

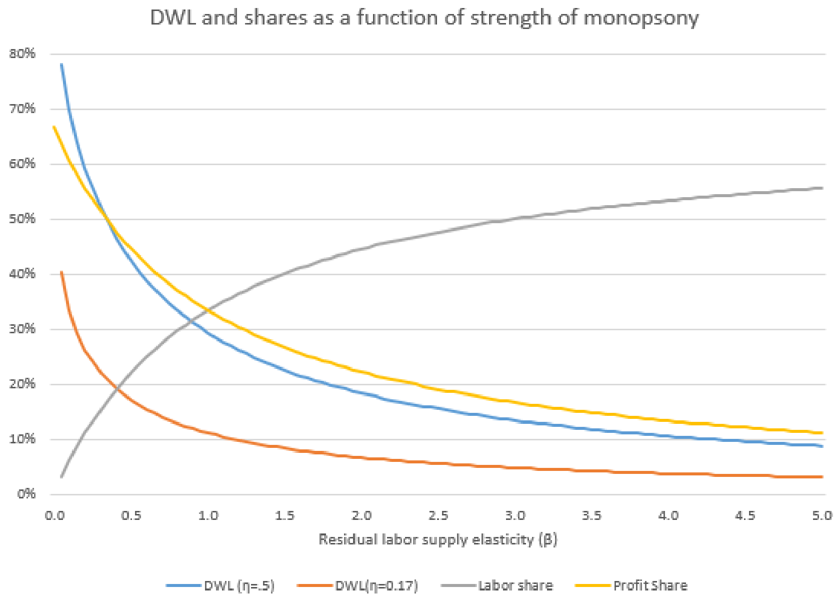

Figure 3: Deadweight loss and labor/profit shares: Cobb-Douglas case

3 CES production, no government

In this section we consider the case of general CES production, without taxes $\tau = 0$. From Equation 5 with $\gamma = 0$ we see that interestingly, the expression for the relative deadweight loss is unchanged from the Cobb-Douglas case considered in the previous section: since $Y^* = \lim_{\beta\to\infty} Y(\beta)$, we get that

$$DWL := \frac{Y^* - Y(\beta)}{Y^*} = 1 - \left(\frac{\beta}{\beta+1}\right)^{\eta}$$

Some logic for why the DWL does not depend on σ comes from the capital/labor ratio expression: $K/L = \left(\frac{\alpha}{1-\alpha}\frac{\beta+1}{\beta}\frac{w}{r}\right)^{\sigma}$. If the wage w changes with β but exactly in such a way that $\tilde{w}$ does not, then the capital labor-ratio is unchanged and output will scale linearly with the labor change brought about by the change in β. Labor itself is iso-elastic in w, so in the CES case–as in the Cobb-Douglas case–overall output falls by the factor $\left(\frac{\beta}{\beta+1}\right)^{\eta}$, which is independent of both α and σ.

Comparing with the CES case with government, the factor $\frac{1+\frac{\beta}{\beta+1}\gamma}{1+\gamma}$ in Equation 5 reflects an interaction between monopsony and the provision of public goods: the relative deadweight loss due to monopsony is exacerbated due to forgone public goods production (which is proportional to γ), in addition to reduced private production. Regardless of the tax rate, both labor provision and private goods production (which is linear in labor after substituting the optimal capital-labor ratio) fall by the factor of $1 - \left(\frac{\beta}{\beta+1}\right)^{\eta}$ due to monopsony power. Since the amount collected by labor tax is proportional to $wL = \frac{\beta}{\beta+1}\tilde{w}L$, government revenue gets an additional factor of $\frac{\beta}{\beta+1}$ in the case with monopsony and falls by a factor $1 - \left(\frac{\beta}{\beta+1}\right)^{\eta+1}$ from the competitive benchmark. In moving from a tax-free to $\tau > 0$ world, although labor and hence output fall by a factor of $(1-\tau)^{\eta}$, total output may still go up if $(1-\tau)^\eta \left(1+\frac{\beta}{\beta+1}\gamma\right) > 1$.

While the DWL is independent of σ, the labor share and profit share do change in our no-tax CES model, compared with the no-tax Cobb-Douglas case. The labor share is

$$\frac{wL}{Y} = \frac{\beta}{\beta+1}\frac{\tilde{w}L}{Y} = \frac{\beta}{\beta+1}(1-\alpha)^\sigma \tilde{w}^{1-\sigma}$$

and the capital share is:

$$\frac{rK}{Y} = \alpha^\sigma r^{1-\sigma}$$

leaving the profit share as

$$\frac{\Pi}{Y} = 1 - \alpha^\sigma r^{1-\sigma} - \frac{\beta}{\beta+1}(1-\alpha)^\sigma \tilde{w}^{1-\sigma}$$

Comparing with the Cobb-Douglas case, the labor share is multiplied by a factor of $\left(\frac{\tilde{w}}{1-\alpha}\right)^{1-\sigma} = \frac{1-\alpha^\sigma r^{1-\sigma}}{1-\alpha}$. With a return on capital of $r = 0.04$, this factor is about 1.36; labor shares are 36% higher than they are under assumption of Cobb-Douglas, without government. In Table 3 and Figure 4, we report results for the deadweight loss and labor/profit shares with $\sigma = 0.4$, and then in Figure 5 we show how the results vary with the elasticity of substitution between capital and labor σ.

Table 3: CES estimates with fixed CES elasticity $\sigma = 0.4$, and $\tau = 0$

β	Implied markdown	β source	**DWL**	Labor share	Profit share
0.1	0.09	Staiger et al. (2010)	**51.3%**	8%	82%
0.12	0.11	Dube et al. (2017a)	**48.8%**	10%	81%
0.95	0.49	Azar et al. (2017)	**19.4%**	44%	46%
2	0.67		**11.5%**	60%	30%
2.7	0.73	Kline et al. (2017)	**9.0%**	66%	25%
3	0.75	Dube et al. (2017b)	**8.3%**	68%	23%
3.7	0.79	Ransom and Sims (2010)	**6.9%**	71%	19%
5.5	0.85	Isen (2013)	**4.9%**	77%	14%
10	0.91		**2.8%**	82%	8%
100	0.99		**0.3%**	90%	1%

4 Cobb-Douglas production with government

Finally, we present results with $\sigma = 1$ but keeping the government taxation and public goods provision in Table 4 and Figure 6. In this case γ has the simpler expression $m\tau(1-\alpha)$, and the expressions from Section 1.2 regarding the deadweight loss, factor, and profit shares can be straightforwardly evaluated at $\sigma = 1$ (the fiscal loss is in fact unchanged, as it does not depend on σ). To labor, the share of output is:

$$\frac{wL+G}{Y+G} = \frac{\beta}{\beta+1}\cdot\frac{(1-\alpha)+\gamma}{1+\frac{\beta}{\beta+1}\gamma}$$

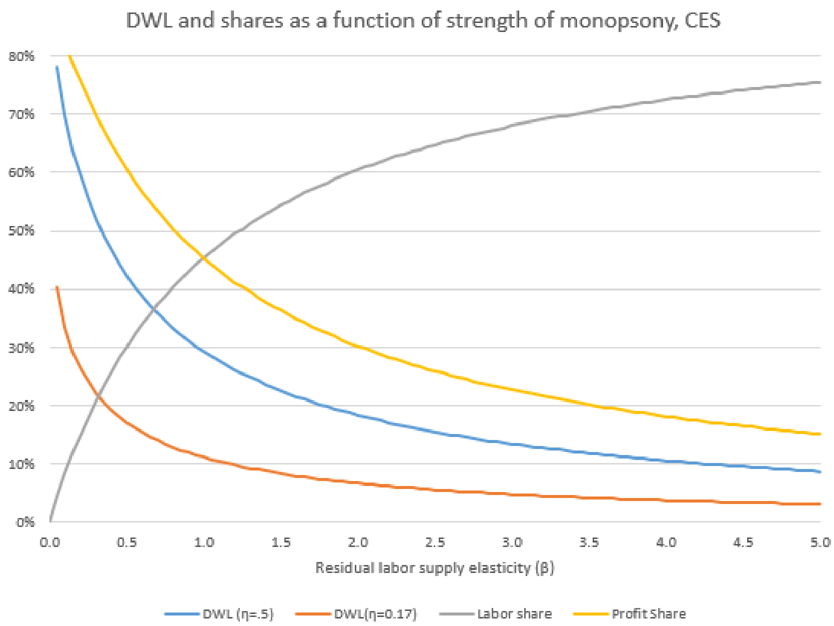

Figure 4: Deadweight loss and labor/profit shares: CES case with $\sigma = 0.4$ and no tax

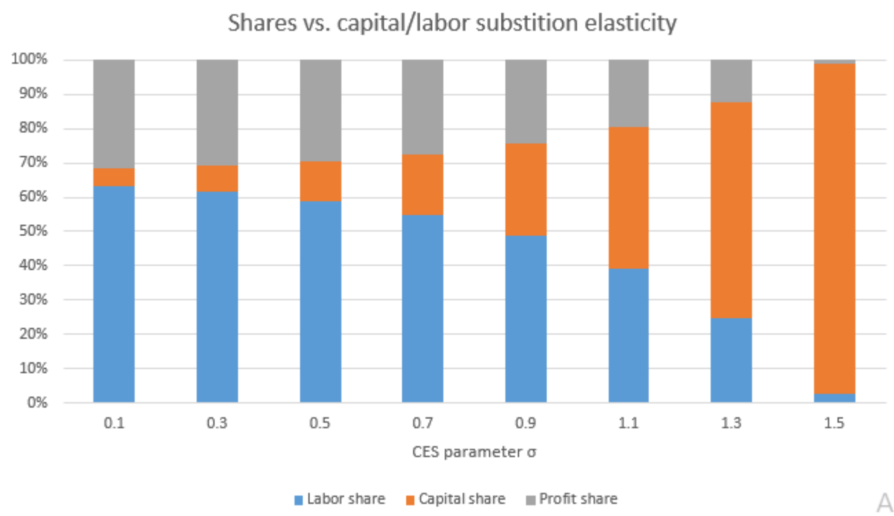

Figure 5: Shares as a function of CES parameter σ, with β fixed at 2 and no tax

To capital:

$$\frac{rK}{Y+G} = \frac{\alpha}{1+\frac{\beta}{\beta+1}\gamma}$$

and to firm profit:

$$\frac{\Pi}{Y+G} = \frac{Y+G-(wL+G)-rK}{Y+G} = \frac{(1-\alpha)/(\beta+1)}{1+\frac{\beta}{\beta+1}\gamma}$$

The comparative static from the CES case we considered ($\sigma = 0.4$) to Cobb-Douglas takes γ from 35% to 26%, reducing the importance of the labor tax and public goods provision in the deadweight loss calculation.

Table 4: Cobb-Douglas estimates under a $\tau = 30\%$ labor tax with efficiency multiplier of $m = 1.3$

β	Implied markdown	β source	**DWL**	Labor share	Profit share	Fiscal loss
0.1	0.09	Staiger et al. (2010)	**60.4%**	8%	59%	96%
0.12	0.11	Dube et al. (2017a)	**58.3%**	10%	58%	95%
0.95	0.49	Azar et al. (2017)	**27.9%**	40%	30%	61%
2	0.67		**17.5%**	53%	19%	41%
2.7	0.73	Kline et al. (2017)	**14.1%**	57%	15%	34%
3	0.75	Dube et al. (2017b)	**13.0%**	58%	14%	31%
3.7	0.79	Ransom and Sims (2010)	**11.0%**	61%	12%	27%
5.5	0.85	Isen (2013)	**7.9%**	64%	8%	20%
10	0.91		**4.6%**	68%	5%	12%
100	0.99		**0.5%**	73%	1%	1%

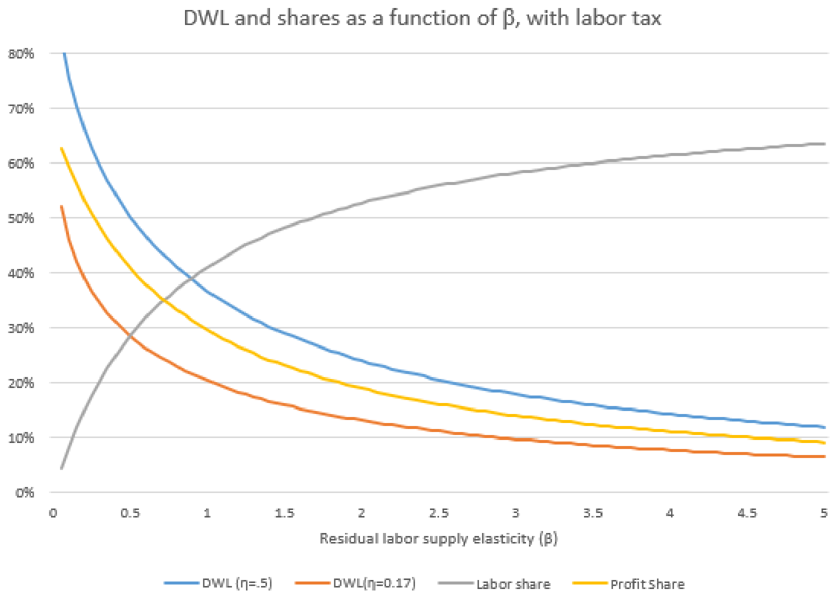

Figure 6: Deadweight loss and labor/profit shares under a $\tau = 30\%$ labor tax with efficiency multiplier of $m = 1.3$, Cobb-Douglas production

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