Evolutionary Bargaining with Intentional Idiosyncratic Play

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Abstract

We introduce intentional idiosyncratic play in a standard stochastic evolutionary model of equilibrium selection in a class of bargaining games. By intentional we mean non-best-response play of mixed strategies that are supported only on the set of strategies that would give the idiosyncratic player a higher payoff were sufficiently many others to do the same. This induces qualitatively different transitions between Nash equilibria and potentially different stochastically stable equilibria than the standard dynamic. We show existence and uniqueness of a stochastically stable bargaining outcome under intentional idiosyncratic play in a class of games that nests contract games and the Nash demand game. In the contract game, the intentional idiosyncratic play dynamic selects the equilibrium that implements the Nash bargain as the stochastically stable state, while the standard dynamic selects the Kalai-Smorodinsky bargain.

Keywords: Evolutionary Game Theory, Stochastic Stability, Nash Bargaining Solution, Multiple Equilibria, Institutional Transitions, Intentionality, Idiosyncratic Play.

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1 Introduction

We extend the Binmore-Samuelson-Young (Binmore et al., 2003) approach to equilibrium selection in contract games and related bargaining games by imposing empirically plausible restrictions on the process generating idiosyncratic (non-best-response) play. (By contract game, (Young, 1998) means an asymmetric pure coordination game played by randomly matched players from two sub-populations.) Our modification to the standard dynamic(Kandori et al., 1993; Young, 1993a) is motivated by our belief that agents who act idiosyncratically in economic conflicts are behaving intentionally, and thus do not "accidentally" experiment with contracts under which they would do worse, should the contract be generally adopted. We have in mind such idiosyncratic play as refusing to exchange under the terms of a contract that awards most of the joint surplus to the other party (for example locking out overly demanding employees). Like Bergin and Lipman (1996), who conclude that "models or criteria to determine 'reasonable' mutation processes should be a focus of research in this area," and Van Damme and Weibull (2002), our idiosyncratic play is state-dependent. But while these authors make error *rates* state dependent, we make the *distribution* of idiosyncratic play across the strategy space state-dependent, as in Bowles (2004).

The resulting dynamic based on intentional idiosyncratic play provides a more plausible account of historical real world transitions between economically important conventions, such as customary crop shares or the de facto recognition of collective bargaining by businesses. First, when non-best-response play is intentional transitions between contracts are induced only by the idiosyncratic play of those who stand to benefit from the switch, in contrast to the standard (unintentional) approach. Second, as one would expect, in the intentional dynamic where sub-population sizes and idiosyncratic play rates differ, the sub-population whose interests are favored is that whose members who engage in more frequent idiosyncratic play and who are less numerous.

We find that the contracts that are selected as stochastically stable under the intentional idiosyncratic play dynamic differ from those selected under the standard dynamic. Our dynamic selects the convention that implements the Nash bargain, while the standard dynamic selects the Kalai-Smorodinsky bargain (Young, 1998; Kalai and Smorodinsky, 1975). The difference is illustrated in the example in Table 1. The Kalai-Smorodinsky bargaining solution equates the ratio of the payoffs to the ratio of the players' best possible payoff, and thus is the contract pair $(1, 1)$, as $12/20 = 36/60$. In contrast, the Nash solution is $(0, 0)$, since the Nash solution is that which maximizes the product of the payoffs and $5 \times 60 > 12 \times 20 > 36 \times 1$.

Table 1: Example 1

Contract	0	1	2
0	5,60	0,0	0,0
1	0,0	12,20	0,0
2	0,0	0,0	36,1

In section 2 we introduce intentional idiosyncratic play, present the main proposition of the paper, and characterize the stochastically stable state under intentional dynamics for a variety of cases.

2 The Model

2.1 Setup

We consider a large population divided into two sub-populations, denoted R and C for row and column, playing an asymmetric bargaining game. This has K strategies, with payoff functions given by $\pi^R(i,i) = a_i, \pi^C(i,i) = b_i$ with $i \in S = \{1, 2, ..., K\}$. We order the strategies such that if $i < j$ then $a_j > a_i$ and $b_j < b_i$, so the contracts are ordered such that the row player favors contracts with higher indices, and the column player favors contracts with lower indices. The off-diagonal payoffs are given by $\pi^R(i,j) = \pi^C(i,j) = 0$ if $i > j$ and $\pi^R(i,j) = \lambda a_i, \pi^C(i,j) = \lambda b_j$ if $i < j$, where $0 \leq \lambda \leq 1$. That is, agents receive some fraction of their demands if the demands together do not exhaust the surplus, and receive 0 otherwise. This formulation excludes some variants of non-cooperative bargaining games, such as the cushioned Nash Demand game, but the contract game (Young, 1998) corresponds to $\lambda = 0$ and the Nash Demand game corresponds to $\lambda = 1$. Clearly the diagonal of the game matrix constitutes the set of pure Nash equilibria, and they are all strict and Pareto-optimal.

The dynamic is a familiar myopic best-response dynamic with inertia. Each period, all players are matched to play the contract game. Each time they are matched, agents revise their strategy with probability v and play the strategy they played last period with probability $1 - v$ and . We represent this dynamic by a stochastic dynamic system, where the states represent the number of each sub-population playing each strategy. The state space is given by $\Xi = \Delta_R \times \Delta_C$,

$$\begin{aligned} \Delta_R &= \{n = (n_1, n_2..., n_K) | \textstyle\sum_i n_i = N\} \\ \Delta_C &= \{m = (m_1, m_2, .., m_K) | \textstyle\sum_i m_i = M\} \end{aligned}$$

where N is the size of the row sub-population and M is the size of the column sub-population, and each n_i and m_i is the number of the row and column sub-population, respectively, that is playing strategy i. Given a state, $(n, m) \in \Xi$ we define best response functions as follows:

$$\begin{aligned} BR_R &: \Delta_C \to S, \quad m \mapsto \arg\max_{i \in S} \sum_{j \in S} \pi_R(i,j) \frac{m_j}{M} \\ BR_C &: \Delta_R \to S, \quad n \mapsto \arg\max_{i \in S} \sum_{j \in S} \pi_C(i,j) \frac{n_j}{N} \end{aligned}$$

where we break ties by choosing the higher indexed strategy. To model intentional idiosyncratic play, we consider a discrete time process indexed by $t = 1, 2, \cdots$. Following Kandori et al. (1993) and Binmore et al. (2003), we define a random best response dynamic:

$$\mathbf{X}_{t+1} = \alpha_t^R \left(Z_{0t}^R \, \mathbf{e}_{BR_R(\mathbf{Y}_t)} + \mathbf{Z}_t^R \right) + (1 - \alpha_t^R) \mathbf{X}_t \tag{1}$$

$$\mathbf{Y}_{t+1} = \alpha_t^C \left(Z_{0t}^C \, \mathbf{e}_{BR_C(\mathbf{X}_t)} + \mathbf{Z}_t^C \right) + (1 - \alpha_t^C) \mathbf{Y}_t \tag{2}$$

where $\mathbf{X}_t = (X_{1t}, X_{2t}, \cdots, X_{Kt})^T$, $\mathbf{Y}_t = (Y_{1t}, Y_{2t}, \cdots, Y_{Kt})^T$, T denotes a transpose, $\mathbf{e}_i$ denotes K dimensional vector with 1 in the i th position and 0 elsewhere. α_t^R, α_t^C are independent Bernoulli random variables taking value 1 with probability ν, which captures inertia. $(Z_{0t}^R, Z_{1t}^R, \cdots, Z_{Kt}^R)$, $(Z_{0t}^C, Z_{1t}^C, \cdots, Z_{Kt}^C)$ are multinomial random variables with N draws and a probability vector τ and we use notations, $\mathbf{Z}_t^R = (Z_{1t}^R, \cdots, Z_{Kt}^R)^T$, $\mathbf{Z}_t^C = (Z_{1t}^C, \cdots, Z_{Kt}^C)^T$. The variables, $\mathbf{Z}_t^R$ and $\mathbf{Z}_t^C$, specify the numbers of agents playing each strategy idiosyncratically, while Z_{0t}^R and Z_{0t}^C represent the numbers of agents playing best responses. When $v = 0$, the dynamic is characterized by full inertia, and we see that $\alpha_t^R = \alpha_t^C = 0$, $\mathbf{X}_{t+1} = \mathbf{X}_t$,and $\mathbf{Y}_{t+1} = \mathbf{Y}_t$. On the other hand in case of $v = 1$, the dynamic is solely driven by the first two terms in (1) and (2). In particular, when

$\tau^R = \tau^C = (1, 0, \cdots, 0)$ so we have $Z^R_{0t} = N$, $Z^C_{0t} = M$ and $\mathbf{Z}^R_t = \mathbf{Z}^C_t = \mathbf{0}$, and the best response functions completely determine the dynamics.

We define τ^R, τ^C which will specify the probability vector for a multinomial random variable, $(Z^R_{0t}, Z^R_{1t}, \cdots, Z^R_{Kt}), (Z^C_{0t}, Z^C_{1t}, \cdots, Z^C_{Kt})$ as:

$$\begin{aligned} \tau^R_i(b) &= \begin{cases} 0 & \text{if } 1 \leq i < b \\ \frac{\epsilon}{K-b+1} & \text{if } b < i \leq K \end{cases} \\ \tau^C_j(b) &= \begin{cases} \frac{\epsilon}{b} & \text{if } 1 \leq j \leq b \\ 0 & \text{if } b < j \leq K \end{cases} \end{aligned} \tag{3}$$

and $\tau^R_0(b) = 1 - \sum_{i=1}^{K} \tau^R_i(b)$ and $\tau^C_0(b) = 1 - \sum_{j=1}^{K} \tau^C_j(b)$, where $\tau^R = (\tau^R_0, \cdots, \tau^R_K)$ and $\tau^C = (\tau^C_0, \cdots, \tau^C_K)$. We write $Z \sim \mathcal{MN}(N, \tau)$ if Z follows a multinomial variable with N draws and a probability vector τ. Now we define an intentional idiosyncratic dynamic. Given a strategy b, we note that $\{i : b \leq i \leq K\}$ is the set of strategies that row population prefers to b because of the indexing of strategies and we consider $\{i : b \leq i \leq K\}$ as a set of strategies from which an idiosyncratic player draws. The idiosyncratic play distribution is state-dependent; row only experiments with strategies which are favorable to the row population.

Definition 1.

- We say $(X_t, Y_t)_{t \in \mathbb{Z}_+}$ in (1) and (2) is an **unperturbed** process if $(Z^R_{0t}, Z^R_t) \sim \mathcal{MN}(N, \tau^R(K+1))$ and $(Z^C_{0t}, Z^C_t) \sim \mathcal{MN}(M, \tau^C(0))$
- We say $(X_t, Y_t)_{t \in \mathbb{Z}_+}$ in (1) and (2) is an **U-process** if $(Z^R_{0t}, Z^R_t) \sim \mathcal{MN}(N, \tau^R(0))$ and $(Z^C_{0t}, Z^C_t) \sim \mathcal{MN}(M, \tau^C(K+1))$
- We say $(X_t, Y_t)_{t \in \mathbb{Z}_+}$ in (1) and (2) is an **I-process** if $(Z^R_{0t}, Z^R_t) \sim \mathcal{MN}(N, \tau^R(BR_R(Y_t)))$ and $(Z^C_{0t}, Z^C_t) \sim \mathcal{MN}(M, \tau^C(BR_C(X_t)))$

It is clear that both the U-process and I-process are finite state space Markov chains and that the transition probability matrix of U-process is irreducible and aperiodic, so the chain admits a unique stationary distribution $\mu(\epsilon)$. We are interested in the stochastically stable states that have positive weight in the distribution μ^*, where $\mu(\epsilon) \to \mu^*$ following Young (1993a). We show that I-process is irreducible and aperiodic in the appendix.

2.2 Unintentional vs Intentional Idiosyncratic Dynamics

The U-process defined above is the standard mutation dynamics encountered in the literature (Kandori et al., 1993; Young, 1993a) . Analyzing the I-process defined above is the contribution of this paper. Binmore et al. (2003) show that the stochastically stable state in the U-process is the Kalai-Smorodinsky Solution in the contract game, and the Nash bargain in the Nash Demand game. It is also useful to describe the transitions between states in the U-process; in the contract game they are driven by mistakes in the population who loses from the transition. Our I-dynamic, in contrast, has agents only erring in the direction that could benefit them; thus the subpopulations driving transitions are the ones that stand to gain. This difference in the relevant population mutations drives the differences in the stochastically stable state that the two processes select.

To analyze the I-dynamic we first identify the state where all individuals in both row population and column population play the same strategy i with contract i; i.e. we identify $(N\mathbf{e}_i, M\mathbf{e}_i)$ with i. Then it is easily seen that i's are the absorbing states in the unperturbed process. Following Binmore et al. (2003) we compute the resistance R_{ij}−minimum number of idiosyncratic players playing k− between these absorbing states, contract i and contract j, in U-process, ignoring integer considerations:

$$R_{ij} = \begin{cases} N\frac{b_i-\lambda b_j}{b_i+(1-\lambda)b_j} & \text{if } i<j \\ M\frac{a_i-\lambda a_j}{a_i+(1-\lambda)a_j} & \text{if } i>j \end{cases}$$

We call trees with R_{ij} edge resistances *I-trees*. From theorem 1 in (Young, 1993a), we know that the I-stable state is contained in the root of the minimal I-tree. In appendix A we show that the U-stable state in example 1 is the Kalai-Smorodinsky solution($a_1/b_1 = a_{max}/b_{max}$), while stochastically stable state under the I dynamic is the Nash Solution($a_0b_0 = max_i a_i b_i$). This is a general difference, as illustrated by the next proposition where we set $b_i = f(a_i)$ and use the notation, $a^*_N = \arg\max_{s\in[0,1]} sf(s)$, and $a^*_G = \arg\max_{s\in[0,1]} s^M(f(s))^N$. We normalize f in a way that $f(0) = 1$ and $f(1) = 0$.

Proposition 2.1. *Suppose the $a_i = i\delta$ and $i \in \{1, ...\frac{1-\delta}{\delta}, \frac{1}{\delta}\}$, where $\delta = \frac{1}{10^n}$ for some positive integer n and f is positive, decreasing, and concave, satisfying $f(0) = 1$ and $f(1) = 0$. Then we have*
(i) If $\lambda \leq 1$, a unique stochastically stable contract in the I-dynamic i^* exists, and is increasing in N/M
(ii) If $\lambda = 1$ and δ is sufficiently small, the stochastically stable contract i^* in the I-dynamic approaches $(a^*_G, f(a^*_G))$.
(iii) If $M = N$ and δ is sufficiently small, the stochastically stable contract i^* in the I-dynamic approaches $(a^*_N, f(a^*_N))$.

Proof. See Appendix B. □

Note that if $\lambda = 1$ (the Nash demand game) the I- and U- dynamics select the same outcome (Young, 1993b). If $N = M$ then the symmetric Nash bargain is I-stable. Note also that if $\lambda \leq 1$ and N is not equal to M, the stochastically stable contract will be closer to the best contract for the group with lower sub-population-size. Smaller groups are favored because the realized level of idiosyncratic play is more likely to exceed the critical level to induce a transition, and in the I dynamic groups benefit from the transitions which their idiosyncratic play induces.

Thus we find that a natural and empirically motivated restriction on idiosyncratic play in bargaining games may select different outcomes, as well as generating an empirically plausible transition dynamic in which smaller group size is an advantage, and groups whose idiosyncratic players induce transitions benefit as a result. For example, $N = M$ and $\lambda = 0$ (Contract game). Then the U-dynamic selects the Kalai-Smorodinsky solution (Young, 1998; Binmore et al., 2003), and the I-dynamic selects the Nash solution. Our I-dynamic is thus another class of bargaining interactions in which a standard result of axiomatic cooperative game theory is replicated by the non-cooperative play of only minimally forward looking individuals with limited information. The contrast between the I-dynamic and the standard model for the contract game illustrates the economic intuitions underlying these results. The key differences result from the fact that in the former transitions are induced by the idiosyncratic play of those who stand to benefit. In the U-dynamic the opposite is the case because it will always take fewer idiosyncratic players to induce best responders to shift to a contract that they prefer over the status quo than to induce them to accommodate a shift to a less advantageous contract. In the U-dynamic, the deviations of one subpopulation induce the other subpopulation to concede, coordinating on a contract that they strictly prefer to the status quo; while in the I-dynamic deviations by one subpopulation must induce the other subpopulation to coordinate on a strictly inferior contract.

3 Appendix A (Not For Publication)

In Table 1, the I-stable contact is 0, while the U-stable contract is 1. Table 2, consisting of tree resistances, which is the sum of transition resistances within each tree, illustrates the calculations for the U-dynamic(3 trees for each root).

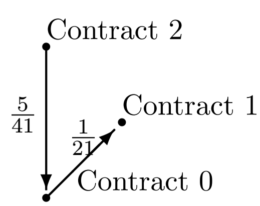
Table 2: U-Resistances for Example 1

Root/Trees			
0	0.266	0.297	0.266
1	0.341	0.310	**0.169**
2	0.544	0.371	0.371

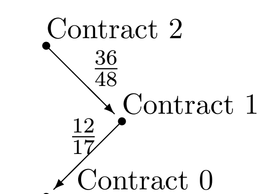
Table 3: I-Resistances for Example 1

Root/Trees			
0	1.583	**1.455**	1.830
1	1.500	1.628	1.733
2	1.702	1.689	1.932

Thus the lowest tree, with resistance $\frac{7}{41}+\frac{1}{21}=0.169$ has root 1. The actual tree is given below.



However, with intentional idiosyncratic play distributions(the I-dynamic), the tree resistances are given in Table 3. The minimal I-tree has root 0, with resistance 1.455, shown in the tree below.



4 Appendix B (Not For Publication)

We first show that I-process is irreducible and aperiodic. This is straightforward, albeit not trivial, since our non-best-responses are not always supported on the entire strategy space. Given an absorbing state $(n, m) \in \Xi$ in the unperturbed process, how can we get to state (n', m') in a finite number of periods? It suffices to point out that we can get to the state $(n_0, m_0) = ((0, 0, ..., N), (M, 0, 0, ..., 0))$, which is where the best responses of both populations are the contract that would be worst for them were it to become an equilibrium, since then the error distribution is supported on the entire state space, and therefore any state is accessible from an absorbing state (n, m). Then since any arbitrary state can reach one of absorbing states, the irreducibility follows. The fact that the chain is aperiodic follows as the inertia of the system implies $\Pr\{(\mathbf{X}_{t+1}, \mathbf{Y}_{t+1}) = (n, m) | (\mathbf{X}_t, \mathbf{Y}_t) = (n, m)\} > 0$ for all n and m. We begin with proofs of parts (ii) and (iii) and then prove part (i).

4.1 Proof of Proposition 2.1 (ii), (iii)

We note that the resistance from state i to j in I-dynamics is

$$R_{ij} = \begin{cases} N\frac{f(a_i)-\lambda f(a_j)}{f(a_i)+(1-\lambda)f(a_j)} & \text{if } i < j \\ M\frac{a_i-\lambda a_j}{a_i+(1-\lambda)a_j} & \text{if } i > j \end{cases} \tag{4}$$

Also from the definition of a_i and the concavity of f, we have $a_i^2 > a_{i-1}a_{i+1}$ and $(f(a_i))^2 > f(a_{i-1})f(a_{i+1})$ for all $i = 2, ..., \frac{1-\delta}{\delta}$. Since we will apply naive minimization test, we establish the following inequalities for each case

$$R_{i,i+1} > R_{i-1,i} \text{ for all } i \tag{5}$$

$$R_{i,i-1} > R_{i+1,i} \text{ for all } i \tag{6}$$

$$R_{ij} < R_{ik} \text{ for all } k > j > i \tag{7}$$

$$R_{ij} < R_{ik} \text{ for all } k < j < i \tag{8}$$

$$R_{i,i+1} < R_{i,i-1} \text{ for all } i < i^* \tag{9}$$

$$R_{i,i-1} < R_{i,i+1} \text{ for all } i > i^* \tag{10}$$

where i^* depends on the case that we prove (defined below). First for both cases (i) and (ii) we observe that (5) and (6) follow from $(f(a_i))^2 > f(a_{i-1})f(a_{i+1})$ and $a_i^2 > a_{i-1}a_{i+1}$ and (7) and (8) follow from the definition of a_i and the fact that f is decreasing.

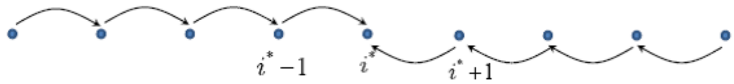
Next we show (9)∼(10) in case (i) of $M = N$. Let $\delta > 0$.Then there exists i_N^* such that $a_{i_N^*}f(a_{i_N^*}) \geq a_i f(a_i)$ for all i. We set $i^* := i_N^*$ in (9) and (10). For $i < i^*$, we have $a_{i+1}f(a_{i+1}) > a_i f(a_i)$. (9) follows from $a_{i+1}f(a_{i+1}) > a_i f(a_i)$, and $a_i^2 > a_{i-1}a_{i+1}$ since in case (i)

$$R_{i,i+1} < R_{i,i-1} \text{ if and only if } a_{i-1}f(a_i) < a_i f(a_{i+1})$$

and

$$\frac{a_i f(a_{i+1})}{a_{i-1}f(a_i)} > \frac{a_i^2}{a_{i-1}a_{i+1}} > 1$$

Similarly when $i > i^*$, we have $a_{i-1}f(a_{i-1}) > a_i f(a_i)$, so (10) follows from $a_{i-1}f(a_{i-1}) > a_i f(a_i)$,and $(f(a_i))^2 > f(a_{i-1})f(a_{i+1})$.



We establish (9)∼(10) in case (ii) of $\lambda = 1$. Again fix $\delta > 0$ and choose i^*_G such that $(a_{i^*_G})^M (f(a_{i^*_G}))^N > (a_i)^M (f(a_i))^N$ for all i. We set $i^* := i^*_G$ in (9) and (10), and define

$$D_\delta f_i := \frac{f(a_{i+1}) - f(a_i)}{\delta}$$

Then we have the following lemma which is proved in the end of proof.

$$\text{for } i < i^*,\ Mf(a_i) + Na_i D_\delta f_i > 0 \tag{11}$$

and

$$\text{for } i > i^*,\ Mf(a_i) + Na_i D_\delta f_{i-1} < 0 \tag{12}$$

Equation (9) follows from (11) and the fact that $R_{i,i+1} < R_{i,i-1}$ if and only if $Na_i D_\delta f_i > -Mf(a_i)$ and similarly equation (10) follows from (12), that $R_{i,i-1} < R_{i,i+1}$ if and only if $Na_i D_\delta f_i < -Mf(a_i)$ in case (2) and $D_\delta f_{i-1} > D_\delta f_i$ (by the concavity of f).
Now (7)∼(10) imply that the naive minimization tree consists of edges in the left of i^* pointing to the right and edges in the right of i^* pointing to the left (see figure below).Also (5)∼(6) shows the tree contains the unique cycle having maximal resistance over all edges. Since $a_{i^*_N}$, $a_{i^*_N+1}$, $a_{i^*_N-1} \to a^*_N$ (case (i)) and $a_{i^*_G}$, $a_{i^*_G+1}$, $a_{i^*_G-1} \to a^*_G$ (case (ii)) as $\delta \to 0$, we conclude the results of the proposition.

Lemma 4.1. *For $i < i^*_G, Mf(a_i) + Na_i D_\delta f_i > 0$ and for $i > i^*_G, Mf(a_i) + Na_i D_\delta f_{i-1} < 0$*

Proof. Let $i < i^*_G$. Then we have $a_i^M (f(a_i))^N < a_{i+1}^M (f(a_{i+1}))^N$, hence $\left(\frac{i+1}{i}\right)^M \left(\frac{f(a_{i+1})}{f(a_i)}\right)^N > 1$. Also since $f(a_{i+1}) = f(a_i) + \delta D_\delta f_i$ implies $\frac{f(a_{i+1})}{f(a_i)} = 1 + \delta \frac{D_\delta f_i}{f(a_i)}$ and $x \geq \log(1+x)$, $x \in \mathbb{R}$, we have

$$\begin{aligned} Mf(a_i) + Na_i D_\delta f_i &= if(a_i)\left(M\frac{1}{i} + N\frac{\delta D_\delta f_i}{f(a_i)}\right) \\ &\geq if(a_i)\left(M\log\left(1+\frac{1}{i}\right) + N\log\left(1+\frac{\delta D_\delta f_i}{f(a_i)}\right)\right) \\ &= if(a_i)\left(M\log\left(1+\frac{1}{i}\right) + N\log\left(\frac{f(a_{i+1})}{f(a_i)}\right)\right) \\ &> 0 \end{aligned}$$

Now let $i > i^*_G$.Then we have $a_{i-1}^M (f(a_{i-1}))^N > a_i^M (f(a_i))^N$ which gives $\left(\frac{i-1}{i}\right)^M \left(\frac{f(a_{i-1})}{f(a_i)}\right)^N > 1$. Also since $\frac{f(a_{i-1})}{f(a_i)} = 1 - \delta \frac{D_\delta f_{i-1}}{f(a_i)}$, we have

$$\begin{aligned} -(Mf(a_i) + Na_i D_\delta f_{i-1}) &= if(a_i)\left(-M\frac{1}{i} - N\frac{\delta D_\delta f_{i-1}}{f(a_i)}\right) \\ &\geq if(a_i)\left(M\log\left(1-\frac{1}{i}\right) + N\log\left(1-\frac{\delta D_\delta f_{i-1}}{f(a_i)}\right)\right) \\ &= if(a_i)\left(M\log\left(1-\frac{1}{i}\right) + N\log\left(\frac{f(a_{i-1})}{f(a_i)}\right)\right) \\ &> 0 \end{aligned}$$

□

4.2 Proof of Proposition 2.1 (i)

Equations (5) $\sim$ (8) still hold for $\lambda \leq 1$ and $M \neq N$. First we define a function ϕ_δ:

$$\phi_\delta(t) := \frac{t-\lambda(t-1)}{t+(1-\lambda)(t-1)} \frac{f(\delta t)+(1-\lambda)f(\delta(t+1))}{f(\delta t)-\lambda f(\delta(t+1))} \text{ for } t \in \mathbb{R}^+ \tag{13}$$

Then it is easily seen that

$$R_{i,i+1} < R_{i,i-1} \text{ if and only if } \frac{N}{M} < \phi_\delta(i)$$

We first note that

$$\phi_\delta(1) = \frac{1+(1-\lambda)f(\delta)}{1-\lambda f(\delta)} > 1 \text{ and} \tag{14}$$

$$\phi_\delta(\frac{1-\delta}{\delta}) = \frac{1-\delta-\lambda(1-2\delta)}{1-\delta+(1-\lambda)(1-2\delta)} < 1 \tag{15}$$

Next we study the sign of derivative of (13)

$$\begin{aligned}\phi'_\delta(t) = & -\frac{1}{(1-2t+(t-1)\lambda)^2 \left(f(\delta_t)-\lambda f(\delta_{t+1})\right)^2} \times \\ & \Bigg[\underbrace{f^2(\delta_t)+(1-2\lambda)f(\delta_t)f(\delta_{t+1})-(1-\lambda)\lambda f^2(\delta_{t+1})}_{I} \\ & +\underbrace{\left\{(2-3\lambda+\lambda^2)t+(-1+4\lambda-2\lambda^2)-\frac{1}{t}\lambda(1-\lambda)\right\}}_{II}\underbrace{\left(f'(\delta_t)f(\delta_{t+1})-f'(\delta_{t+1})f(\delta_t)\right)}_{III}\delta t\Bigg]\end{aligned}$$

where we use notations $\delta_t := \delta t$, $\delta_{t+1} := \delta(t+1)$. Then using $f(\delta_t) > f(\delta_{t+1})$, we have

$$\begin{aligned} I &= f^2(\delta_t)+(1-2\lambda)f(\delta_t)f(\delta_{(t+1)})-(1-\lambda)\lambda f^2(\delta_{t+1}) \\ &> f(\delta_t)f(\delta_{t+1})+(1-2\lambda)f(\delta_t)f(\delta_{t+1})-(1-\lambda)\lambda f^2(\delta_{t+1}) \\ &> 2(1-\lambda)f(\delta_t)f(\delta_{t+1})-(1-\lambda)\lambda f^2(\delta_{t+1}) \\ &> (1-\lambda)f^2(\delta_{t+1}) \\ &> 0 \end{aligned}$$

Next for $t \geq 1$, since $2-3\lambda+\lambda^2 > 0$

$$\begin{aligned} II &= (2-3\lambda+\lambda^2)t+(-1+4\lambda-2\lambda^2)-\frac{1}{t}\lambda(1-\lambda) \\ &> (2-3\lambda+\lambda^2)+(-1+4\lambda-2\lambda^2)-\lambda(1-\lambda) \\ &> 0 \end{aligned}$$

Finally from the fact that f is decreasing and concave, we have $-f'(\delta_{t+1}) > -f'(\delta_t) > 0$ and so $-f'(\delta_{t+1})f(\delta_t) > -f'(\delta_t)f(\delta_t) > -f'(\delta_t)f(\delta_{t+1})$. Thus

$$III = f'(\delta_t)f(\delta_{t+1})-f'(\delta_{t+1})f(\delta_t) > 0$$

so we find $\phi'_\delta(t) < 0$ for $t > 1$.Therefore from (14), (15) and $\phi'_\delta(t) < 0$,there exists a unique t^* such that

$$\text{for } t < t^*,\ \frac{N}{M} < \phi_\delta(t), \text{ and for } t > t^*,\ \frac{N}{M} > \phi_\delta(t)$$

and t^* increases as M increases and decrease as N increases. The existence and properties of i^* in the proposition follow from the existence and properties of t^*.

References

Bergin, J., Lipman, B., 1996. Evolution with state dependent mutations. Econometrica 70, 281–297.

Binmore, K., Samuelson, L., Young, H. P., 2003. Equilibrium selection in bargaining models. Games and Economic Behavior 46, 296–328.

Bowles, S., 2004. Microeconomics: Behavior, Institutions, and Evolution. Princeton University Press, Princeton, NJ.

Kalai, E., Smorodinsky, M., 1975. Other solutions to nash's bargaining problem. Econometrica 43 (3), 513–518.

Kandori, M. G., Mailath, G., Rob, R., 1993. Learning, mutation, and long run equilibria in games. Econometrica 61, 29–56.

Van Damme, E., Weibull, J., 2002. Evolution in games with endogenous mistake probabilities. Journal of Economic Theory 106, 296–315.

Young, H. P., Jan. 1993a. The evolution of conventions. Econometrica 61 (1), 57–84.

Young, H. P., Feb. 1993b. An evolutionary model of bargaining. Journal of Economic Theory 59 (1), 145–168.

Young, H. P., 1998. Conventional contracts. Review of Economic Studies 65, 776–792.