Introduction to the volume

The Economy as an Evolving Complex System II,

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Introduction: Process and Emergence in the Economy

by W. Brian Arthur, Steven Durlauf and David A. Lane.

In September 1987 twenty people came together at the Santa Fe Institute to talk about "the economy as a evolving, complex system." Ten were theoretical economists, invited by Kenneth J. Arrow, and ten were physicists, biologists and computer scientists, invited by Philip W. Anderson. The meeting was motivated by the hope that new ideas bubbling in the natural sciences, loosely tied together under the rubric of "the sciences of complexity," might stimulate new ways of thinking about economic problems. For ten days, economists and natural scientists took turns talking about their respective worlds and methodologies. While physicists grappled with general equilibrium analysis and noncooperative game theory, economists tried to make sense of spin glass models, Boolean networks, and genetic algorithms.

The meeting left two legacies. The first was a volume of essays, *The Economy as an Evolving Complex System*, edited by Arrow, Anderson and David Pines. The other was the founding, in 1988, of the Economics Program at the Santa Fe Institute, the Institute's first resident research program. The Program's mission was to encourage the understanding of economic phenomena from a complexity perspective, which involved the development of theory as well as tools for modeling and for empirical analysis. To this end, since 1988, the Program has brought researchers to Santa Fe, sponsored research projects, held several workshops each year, and published several dozen working papers. And since 1994, it has held an annual summer school for economics graduate students.

This volume, *The Economy as an Evolving Complex System II*, represents the proceedings of an August, 1996 workshop sponsored by the SFI Economics Program. The intention of this workshop was to take stock, to ask: What has a complexity perspective contributed to economics in the past decade? In contrast to the 1987 workshop, almost all of the presentations addressed economic problems, and most presenters were economists by training. In addition, while some of the work presented was conceived or carried out at the Institute, some of the participants had no previous relation with SFI—research related to the complexity perspective is under active development now in a number of different institutes and university departments.

But just what is the complexity perspective in economics? That is not an easy question to answer.

Its meaning is still very much under construction, and in fact the present volume is intended to contribute to that construction process. Indeed, the authors of the essays in this volume by no means share a single, coherent vision of the meaning and significance of complexity in economics. What we will find instead is a family resemblance, based upon an interrelated set of themes that together constitute the current meaning of the complexity perspective in economics.

Several of these themes, already active subjects of research by economists in the mid-1980s, are well described in the earlier *Economy as an Evolving Complex System*: in particular, applications of nonlinear dynamics to economic theory and data analysis, surveyed in the 1987 meeting by Michele Boldrin and William Brock; and the theory of positive feedback and its associated phenomenology of path-dependence and lock-in, discussed by W. Brian Arthur. Research related to both these themes has flourished since 1987, both in and outside the SFI Economics Program. While chaos has been displaced from its 1987 place at center stage of the interest in nonlinear dynamics, in the last decade economists have made substantial progress in identifying patterns of nonlinearity in financial time series and in proposing models that both offer explanations for these patterns and help to analyze and even to some extent predict the series in which they are displayed. Brock surveys both these developments in his chapter in this volume, while positive feedback plays a central role in the models analyzed by Lane (on information contagion), Durlauf (on inequality) and Krugman (on economic geography), and lurk not far under the surface of the phenomena described by North (development) and Leijonhufvud (high inflation).

Looking back over the developments in the past decade, and of the papers produced by the program, we believe that a coherent perspective—sometimes called the "Santa Fe approach"—has emerged within economics. We will call this the complexity perspective, or Santa Fe perspective, or occasionally the process-and-emergence perspective. Before we describe this, we first sketch the two conceptions of the economy that underlie standard, neoclassical economics (and indeed most of the presentations by economic theorists at the earlier, 1987 meeting). We can call these conceptions the "equilibrium" and "dynamical systems" approaches. In the equilibrium approach, the problem of interest is to derive, from the rational choices of individual optimizers, aggregate-level "states of the economy" (prices in general equilibrium analysis, a set of strategy assignments in game theory with associated payoffs) that satisfy some aggregate-level consistency condition (market-clearing, Nash equilibrium), and to examine the properties of these aggregate-level states. In the dynamical systems approach, the state of the economy is represented by a set of variables, and a system of difference equations or differential equations describes how these variables change over time. The problem is to examine the resulting trajectories, mapped over the state space.

However, the equilibrium approach does not describe the *mechanism* whereby the state of the economy changes over time—nor indeed how an equilibrium comes into being.¹ And the dynamical system approach generally fails to accommodate the distinction between *agent-* and *aggregate-*levels (except by obscuring it through the device of "representative agents"). Neither accounts for the emergence of new kinds of relevant state variables, much less new entities, new patterns, new structures.²

To describe the complexity approach, we begin by pointing out six features of an economy that together present difficulties for the traditional mathematics used in economics:³

- **Dispersed Interaction** What happens in the economy is determined by the interaction of many dispersed, possibly heterogeneous, agents acting in parallel. The action of any given agent depends upon the anticipated actions of a limited number of other agents and on the aggregate state these agents co-create
- **No Global Controller** No global entity controls interactions. Instead, controls are provided by mechanisms of competition and coordination between agents. Economic actions are mediated by legal institutions, assigned roles, and shifting associations. Nor is there a universal competitor—a single agent that can exploit all opportunities in the economy
- Cross-cutting Hierarchical Organization The economy has many levels of organization and interaction. Units at any given level—behaviors, actions, strategies, products—typically serve as 'building blocks' for constructing units at the next higher level. The overall organization is more than hierarchical, with many sorts of tangling interactions (associations, channels of communication) across levels
- **Continual Adaptation** Behaviors, actions, strategies, and products are revised continually as the individual agents accumulate experience—the system constantly adapts
- **Perpetual Novelty** Niches are continually created by new markets, new technologies, new behaviors, new institutions. The very act of filling a niche may provide new niches. The result is ongoing, perpetual novelty.

¹ Since an *a-priori* intertemporal equilibrium hardly counts as a mechanism.

² Norman Packard's contribution to the 1987 meeting addresses just this problem with respect to the dynamical systems approach. As he points out, "if the set of relevant variables changes with time, then the state space is itself changing with time, which is not commensurate with a conventional dynamical systems model."

³ John Holland's outline at the 1987 meeting beautifully—and presciently—frames these features. For an early description of the Santa Fe approach, see also the program's 1989 newsletter, "Emergent Structures."

Out-of-Equilibrium Dynamics Because new niches, new potentials, new possibilities, are continually created, the economy operates far from any optimum or global equilibrium. Improvements are always possible and indeed occur regularly.

Systems with these properties have come to be called *adaptive nonlinear networks*. (The term is John Holland's, 1987.) There are many such in nature and society: nervous systems, immune systems, ecologies, as well as economies. An essential element of adaptive nonlinear networks is that they do not act simply in terms of stimulus and response. Instead they anticipate. In particular, economic agents form expectations—they build up models of the economy and act on the basis of predictions generated by these models. These anticipative models need neither be explicit, nor coherent, nor mutually consistent.

Because of the difficulties outlined above, the mathematical tools economists customarily use, which exploit linearity, fixed points, and systems of differential equations, cannot provide a deep understanding of adaptive nonlinear networks. Instead, what is needed is new classes of combinatorial mathematics and population-level stochastic processes, in conjunction with computer modeling. These mathematical and computational techniques are in their infancy. But they emphasize the *discovery* of structure and the *processes* through which structure *emerges* across different levels of organization.

This conception of the economy as an adaptive nonlinear network—an evolving, complex system—has profound implications for the foundations of economic theory and for the way in which theoretical problems are cast and solved. We interpret these implications as follows:

Cognitive foundations Neoclassical economic theory has a unitary cognitive foundation: economic agents are rational optimizers. This means that (in the usual interpretation) agents evaluate uncertainty probabilistically, revise their evaluations in the light of new information via Bayesian updating, and choose the course of action that maximizes their expected utility. As glosses on this unitary foundation, agents are generally assumed to have common knowledge about each other and rational expectations about the world they inhabit (and of course co-create). In contrast, the Santa Fe viewpoint is pluralistic. Following modern cognitive theory, we posit no single, dominant mode of cognitive processing. Rather, we see agents as having to cognitively structure the problems they face—as having to "make sense" of their problems—as much as solve them. And they have to do this with cognitive resources that are limited. To "make sense," to learn, and to adapt, agents use variety of distributed cognitive processes. The very categories agents use

to convert information about the world into action emerge from experience, and these categories or cognitive props need not fit together coherently in order to generate effective actions. Agents therefore inhabit a world that they must cognitively interpret—one that is complicated by the presence and actions of other agents and that is ever changing. It follows that agents generally do not optimize in the standard sense, not because they are constrained by finite memory or processing capability, but because the very concept of an optimal course of action often cannot be defined. It further follows that the deductive rationality of neoclassical economic agents occupies at best a marginal position in guiding effective action in the world. And it follows that any "common knowledge" agents might have about one another must be attained from concrete, specified cognitive processes operating on experiences obtained through concrete interactions. Common knowledge cannot simply be assumed into existence.

Structural foundations In general equilibrium analysis, agents do not interact with one another directly, but only through impersonal markets. By contrast in game theory, all players interact with all other players, with outcomes specified by the game's payoff matrix. So interaction structures are simple and often extreme—one-with-all or all-with-all. Moreover, the internal structure of the agents themselves is abstracted away.⁴ In contrast, from a complexity perspective, structure matters. First, network-based structures become important. All economic action involves interactions among agents, so economic functionality is both constrained and carried by networks defined by recurring patterns of interaction among agents. These network structures are characterized by relatively sparse ties. Second, economic action is structured by emergent social roles and by socially-supported procedures—that is, by institutions. Third, economic entities have a recursive structure: they are themselves comprised of entities. The resulting "level" structure of entities and their associated action processes is not strictly hierarchical, in that component entities may be part of more than one higher-level entity and entities at multiple levels of organization may interact. Thus reciprocal causation operates between different levels of organization—while action processes at a given level of organization may sometimes by viewed as autonomous, they are nonetheless constrained by action patterns and entity structures at other levels. And they may even give rise to new patterns and entities at both higher and lower levels. From the Santa Fe perspective, the fundamental principle of organization is the idea of that units at one level combine to produce units at the next higher level.⁵

⁴ Except in principal-agent theory or transaction-costs economics, where a simple hierarchical structure is supposed to obtain.

⁵ We need not commit ourselves to what constitutes economic "units" and "levels". This will vary from problem context to problem context.

What counts as a problem and as a solution It should be clear by now that exclusively posing economic problems as multi-agent optimization exercises makes little sense from the viewpoint we are outlining—a viewpoint that puts emphasis on process, not just outcome. In particular, it asks how new "things" arise in the world—cognitive things, like "internal models"; physical things, like "new technologies"; social things, like new kinds of economic "units." And it is clear that if we posit a world of perpetual novelty, then outcomes cannot correspond to steadystate equilibria, whether Walrasian, Nash, or dynamic-systems-theoretical. The only descriptions that can matter in such a world are about transient phenomena—about process and about emergent structures. What then can we know about the economy from a process-and-emergence viewpoint, and how can we come to know it? Studying process and emergence in the economy has spawned a growth industry in the production of what are now generally called "agent-based models." And what counts as a solution in an agent-based model is currently under negotiation. Many of the papers in this volume—including those by Arthur et al, Darley and Kauffman, Shubik, Lindgren, Kollman et al, Kirman, and Tesfatsion—address this issue, explicitly or implicitly. We can characterize these as seeking emergent structures arising in interaction processes, in which the interacting entities anticipate the future through cognitive procedures that themselves involve interactions taking place in multi-level structures.

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A description of an approach to economics, however, is not a research program. To build a research program around a process-and-emergence perspective, two things have to happen. First, concrete economic problems have to be identified for which the approach may provide new insights. A number of candidates are offered in this volume: artifact innovation (Lane and Maxfield), the evolution of trading networks (Ioannides, Kirman and Tesfatsion), money (Shubik), the origin and spatial distribution of cities (Krugman), asset pricing (Arthur et al., Brock), high inflation (Leijonhuvfud), persistent differences in income between different neighborhoods or countries (Durlauf). Second, cognitive and structural foundations for modeling these problems have to be constructed, and methods developed for relating theories based on these foundations to observable phenomena (Manski). Here, while substantial progress has been made since 1987, the program is far from complete.

The essays in this volume describe a series of parallel explorations of the central themes of process and emergence in an interactive world—of how to study systems capable of generating perpetual

novelty. These explorations do not form a coherent whole. They are sometimes complementary, sometimes even partially contradictory. But what could be more appropriate to the Santa Fe perspective, with its emphasis on distributed processes, emergence, and self-organization? Here are our interpretations of the research directions that seem to be emerging from this process:

Cognition The central cognitive issues raised in this volume are ones of interpretation. As Shubik puts it, "the interpretation of data is critical. It is not what the numbers are, but what they mean." How do agents render their world comprehensible enough so that "information" has meaning? The two papers by Arthur, Holland, LeBaron, Palmer and Tayler and by Darley and Kauffman consider this. They explore problems in which a group of agents take actions whose effects depend on what the other agents do. The agents base their actions on expectations they generate about how other agents will behave. Where do these expectations come from? Both papers reject common knowledge or common expectations as a starting point. Indeed, Arthur et al. argue that common beliefs cannot be deduced. Because agents must derive their expectations from an imagined future that is the aggregate result of other agents' expectations, there is a self-reference of expectations that leads to deductive indeterminacy. Rather, both papers suppose that each agent has access to a variety of "interpretative devices" that single out particular elements in the world as meaningful and suggest useful actions on the basis of the "information" these elements convey. Agents keep track of how useful these devices turn out to be, discarding ones that produce bad advice and tinkering to improve those that work. In this view, economic action arises from an evolving ecology of "interpretive devices" that interact with one another through the medium of the agents that use them to generate their expectations.

Arthur *et al.* build a theory of asset pricing upon such a view. Agents—investors—act as market statisticians. They continually generate expectational models—interpretations of what moves prices in the market —and test these by trading. They discard and replace models if not successful. Expectations in the market therefore become endogenous—they continually change and adapt to a market that they together create. The Arthur *et al.* market settles into a rich psychology, in which speculative bubbles, technical trading and persistence of volatility emerge. The homogeneous rational expectations of the standard literature become a special case—possible in theory but unlikely to emerge in practice. Brock presents a variant of this approach, allowing agents to switch between a limited number of expectational models. His model is simpler than that of Arthur *et al.*, but he achieves analytical results, which he relates to a variety of stylized facts about financial times series, many of which have been uncovered through the application of nonlinear analysis over the past decade.

In the world of Darley and Kauffman, agents are arrayed on a lattice, and they try to predict the behavior of their lattice neighbors. They generate their predictions via an autoregressive model, and they can individually tune the number of parameters in the model and the length of the time series they use to estimate model parameters. Agents can change parameter number or history length by steps of length 1 each period, if by doing so they would have generated better predictions in the previous period. This induces a coevolutionary "interpretative dynamics," which does not settle down to a stable regime of precise, coordinated mutual expectations. In particular, when the system approaches a "stable rational-expectations state," it tends to break down into a disordered state. They use their results to argue against conventional notions of rationality, with infinite foresight horizons and unlimited deductive capability.

In his paper on high inflation, Leijonhufvud poses the same problem as Darley and Kauffman: Where should we locate agent cognition, between the extremes of "infinite-horizon optimization" and "myopic adaptation"? Leijonhufvud argues that the answer to this question is context dependent. He claims that in situations of institutional break-down like high inflation, agent cognition shifts toward the "short memory/short foresight adaptive mode." The causative relation between institutional and cognitive shifts becomes reciprocal. With the shrinking of foresight horizons, markets for long-term loans (where long-term can mean over 15 days) disappear. And as inflation accelerates, units of accounting lose meaning. Budgets cannot be drawn in meaningful ways, the executive arm of government becomes no longer fiscally accountable to parliament, and local governments become unaccountable to national governments. Mechanisms of social and economic control erode. Ministers lose control over their bureaucracies, shareholders over corporate management.

The idea that "interpretative devices" such as explicit forcasting models and technical-trading rules play a central role in agent cognition fits with a more general set of ideas in cognitive science, summarized in Clark (1996). This work rejects the notion that cognition is all "in the head." Rather, interpretive aids such as autoregressive models, computers, languages or even navigational tools (as in Hutchins, 1995) and institutions provide a "scaffolding," an external structure on which much of task of interpreting the world is off-loaded. Clark (1996) argues that the distinctive hallmark of in-the-head cognition is "fast pattern completion," which bears little relation to the neoclassical economist's deductive rationality. In this volume, North takes up this theme, describing some of the ways in which institutions scaffold interpretations of what constitutes possible and appropriate action for economic agents.

Lane and Maxfield consider the problem of interpretation from a different perspective. They are particularly interested in what they call attributions of functionality: interpretations about what an artifact does. They argue that new attributions of functionality arise in the context of particular kinds of agent relationships, where agents can differ in their interpretations. As a consequence, cognition has an unavoidable social dimension. What interpretations are possible depend on who interacts with whom, about what. They also argue that new functionality attributions cannot be foreseen outside the particular generative relationships in which they arise. This unforeseeability has profound consequences for what constitutes "rational" action in situations of rapid change in the structure of agent-artifact space.

All the papers mentioned so far take as fundamental the importance of cognition for economic theory. But the opposite point of view can also be legitimately defended from a process-andemergence perspective. According to this argument, overrating cognition is just another error deriving from methodological individualism, the very bedrock of standard economic theory. How individual agents decide what to do may not matter very much. What happens as a result of their actions may depend much more on the interaction structure through which they act—who interacts with whom, according to which rules. Blume makes this point in the introduction to his paper on population games, which, as he puts it, provide a class of models that shift attention "from the fine points of individual-level decision theory to dynamics of agent interaction." Padgett makes a similar claim, though for a different reason. He is interested in formulating a theory of the firm as a locus of transformative "work," and he argues that "work" may be represented by "an orchestrated sequence of actions and reactions, the sequence of which produces some collective result (intended or not)." Hence, studying the structure of coordinated action-reaction sequences may provide insight into the organization of economic activity, without bringing "cognition" into the story at all. Padgett's paper is inspired by recent work in chemistry and biology (by Eigen and Schuster and by Fontana and Buss, among others) that are considered exemplars of the complexity perspective in these fields.

Structure Most human interactions, even those taking place in "economic" contexts, have a primarily social character: talking with friends, asking advice from knowledgeable acquaintances, working together with colleagues, living next to neighbors. Recurring patterns of such social interactions bind agents together into networks.⁶ According to standard economic theory, what

⁶ There is a voluminous sociological literature on interaction networks. Recent entry points include Noria and Eccles (1992), particularly the essay by Granovetter entitled "Problems of Explanation in Economic Sociology", and the methodological survey of Wasserman and Faust (1994).

agents do depends on their values and available information. But standard theory typically ignores where values and information come from. It treats agents' values and information as exogenous and autonomous. In reality, agents learn from each other, and their values may be influenced by others' values and actions. These processes of learning and influencing happen through the social interaction networks in which agents are embedded, and they may have important economic consequences. For example, one of the models presented in Durlauf's paper implies that value relationships among neighbors can induce persistent income inequalities between neighborhoods. Lane examines a model in which information flowing between agents in a network determines the market shares of two competing products. Kirman's paper reviews a number of models that derive economic consequences from interaction networks.

Ioannides, Kirman and Tesfatsion consider the problems of how networks emerge from initially random patterns of dyadic interaction and what kinds of structure the resulting networks exhibit. Ioannides studies mathematical models based on controlled random fields, while Tesfatsion works in the context of a particular agent-based model, in which the "agents" are strategies that play Prisoner's Dilemma with one another. Ioannides and Tesfatsion are both primarily interested in networks involving explicitly economic interactions, in particular trade. Their motivating idea, long recognized among sociologists (for example, Baker, 1984), is that markets actually function by means of networks of traders, and what happens in markets may reflect the structure of these networks, which in turn may depend on how the networks emerge.

Local interactions can give rise to large-scale spatial structures. This phenomenon is investigated by several of the papers in this volume. Lindgren's contribution is particularly interesting in this regard. Like Tesfatsion, he works with an agent-based model in which the agents code strategies for playing 2-person games. In both Lindgren's and Tesfatsion's models, agents adapt their strategies over time in response to their past success in playing against other agents. Unlike Tesfatsion's agents, who meet randomly and decide whether or not to interact, Lindgren's agents only interact with neighbors in a prespecified interaction network. Lindgren studies the emergence of spatio-temporal structure in agent space—metastable ecologies of strategies that maintain themselves for many agent-generations against "invasion" by new strategy types or "competing" ecologies at their spatial borders. In particular, he compares the structures that arise in a lattice network, in which each agent interacts with only a few other agents, and with those that arise in a fully-connected network, in which each agent interacts with all other agents. He finds that the former "give rise to a stable coexistence between strategies that would otherwise be outcompeted. These spatio-temporal structures may take the form of spiral waves, irregular waves, spatio-

temporal chaos, frozen patchy patterns, and various geometrical configurations." Though Lindgren's model is not explicitly economic, the contrast he draws between an agent space in which interactions are structured by (relatively sparse) social networks and an agent space in which all interactions are possible (as is the case, at least in principle, with the impersonal markets featured in general equilibrium analysis) is suggestive. Padgett's paper offers a similar contrast, in a quite different context.

Both Durlauf and Krugman explore the emergence of geographical segregation. In their models, agents may change location—that is, change their position in a social structure defined by neighbor ties. In these models (especially Durlauf's), there are many types of agents, and the question is under what circumstances, and through what mechanisms, do aggregate-level "neighborhoods" arise, each consisting predominantly (or even exclusively) of one agent type. Thus, agent's choices, conditioned by current network structure (the agent's neighbors and the neighbors at the sites to which the agent can move), changes that structure; over time, from the changing local network structure, an aggregate-level pattern of segregated neighborhoods emerges.

Kollman, Miller and Page explore a related theme in their work on political platforms and institutions in multiple jurisdictions. In their agent-base model, agents may relocate between jurisdictions. They show that when their are more than three jurisdictions, two-party competition outperforms democratic referenda. The opposite is the case when there is only one jurisdiction and hence no agent mobility. They also find that two-party competition results in more agent moves than does democratic referenda.

Manski reminds us that while theory is all very well, understanding of real phenomena is just as important. He distinguishes between three kinds of causal explanation for the often observed empirical fact that "persons belonging to the same group tend to behave similarly." One is the one we have been describing above: the behavioral similarities may arise through network interaction effects. But there are two other possible explanations: *contextual*, in which the behavior may depend on exogenous characteristics of the group (like socioeconomic composition); or *correlated effects*, in which the behavior may be due to similar *individual* characteristics of members of the group. Manski shows, among other results, that a researcher who uses the popular linear-in-means model to analyze his data and "observes equilibrium outcomes and the composition of reference groups cannot empirically distinguish" endogenous interactions from these alternative explanations. One moral is that nonlinear effects require nonlinear inferential techniques.

In the essays of North, Shubik and Leijonhufvud, the focus shifts to another kind of social structure, the institution. North's essay focuses on institutions and economic growth, Shubik's on financial institutions, and Leijonhufvud on high inflation phenomenology. All three authors agree in defining institutions as "the rules of the game," without which economic action is unthinkable. They use the word "institution" in at least three senses: as the "rules" themselves (for example, bankruptcy laws); as the entities endowed with the social and political power to promulgate rules (for example, governments and courts); and as the socially legitimized constructions that instantiate rules and through which economic agents act (for example, fiat money and markets). In whichever sense institutions are construed, the three authors agree that they cannot be adequately understood from a purely economic, purely political, or purely social point of view. Economics, politics and society are inextricably mixed in the processes whereby institutions come into being. And they change, and determine economic, political and social action. North also insists that institutions have a cognitive dimension, through the aggregate-level "belief systems" that sustain them and determine the directions in which they change.

North takes up the question of the emergence of institutions from a functionalist perspective: institutions are brought into being "in order to reduce uncertainty," that is, to make agents' worlds predictable enough to afford recognizable opportunities for effective action. In particular, modern economies depend upon institutions that provide low transaction costs in impersonal markets.

Shubik takes a different approach. His analysis starts from his notion of strategic market games. These are "fully defined process models" that specify actions "for all points in the set of feasible outcomes." He shows how, in the context of constructing a strategic market game for an exchange economy using fiat money, the full specification requirement leads to the logical necessity of certain kinds of rules that Shubik identifies with financial institutions. Geanakoplos' paper makes a similar point to Shubik's. Financial instruments represent promises, he argues. What happens if someone cannot or will not honor a promise? Shubik already introduced the logical necessity of one institution, bankruptcy law, to deal with defaults. Geanakoplos introduces another, collateral. He shows that, in equilibrium, collateral as an institution has institutional implications—missing markets.

Finally, in his note concluding the volume, Philip Anderson provides a physicist's perspective on a point that Fernand Braudel argues is a central lesson from the history of long-term socioeconomic change. Averages and assumptions of agent homogeneity can be very deceptive in complex systems. And processes of change are generally driven by the inhabitants of the extreme tails of

some relevant distribution. Hence an interesting theoretical question from the complexity perspective is: How do distributions with extreme tails arise, and why are they so ubiquitous and so important?

What counts as a problem and as a solution While the papers here have much to say on cognition and structure, they contain much less discussion on what constitutes a problem and solution from this new viewpoint. Perhaps this is because it is premature to talk about methods for generating and assessing understanding when what is to be understood is still under discussion. While a few of the papers completely avoid mathematics, most of the papers do present mathematical models—whether based on statistical mechanics, strategic market games, random graphs, population games, stochastic dynamics, or agent-based computations. Yet sometimes the mathematical models the authors use leave important questions unanswered. For example, in what way do equilibrium calculations provide insight into emergence? This troublesome question is not addressed in any of the papers, even those in which models are presented from which equilibria are calculated—and insight into emergence is claimed to result. Blume raises two related issues in his discussion of population games: whether the asymptotic equilibrium selection theorems featured in the theory happen "soon enough" to be economically interesting; and whether the invariance of the "global environment" determined by the game and interaction model is compatible with an underlying economic reality in which rules of the game undergo endogenous change. It will not be easy to resolve the inherent tension between traditional mathematical tools and phenomena that may exhibit perpetual novelty.

As we mentioned previously, several of the papers introduce less traditional, agent-based models. Kollman, Miller and Page discuss both advantages and difficulties associated with this set of techniques. They end up expressing cautious optimism about their future usefulness. Tesfatsion casts her own paper as an illustration of what she calls "the alife approach for economics, as well as the hurdles that remain to be cleared." Perhaps the best recommendation we can make to the reader with respect to the epistemological problems associated with the process-and-emergence perspective is simple. Read the papers, and see what you find convincing.