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COMBINATORIAL EVOLUTION

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Abstract

This chapter discusses a mechanism that lies behind evolution in a number of systems: combinatorial evolution. In this mechanism, novel elements are created by combining earlier existing elements, and they become available as building blocks for creating yet further novel elements. This defines a mechanism of heredity: The system evolves. Combinatorial evolution was introduced as a concept in Arthur (2009) and it since has received widespread attention. This chapter examines how combinatorial evolution works and shows that it occurs naturally in a class of systems that includes technology, chemistry, mathematics, computation, language, and parts of biology.

In this chapter I want to talk about a mechanism that lies behind evolution[1] in a number of systems: direct combination. Novel elements can emerge from the combination of pre-existing elements, and become available as potential building blocks for creating yet further elements. The system has a means of heredity—it evolves. Chloroplasts, the small but essential organelles in plant cells responsible for photosynthesis, are believed to have originated from cyanobacteria—photosynthetic bacteria that were once free-living—combining with primitive

[1] Evolution, according to Stephen J. Gould (1989), has two main meanings: that "lineages alter their form and diversity through time by a natural process of change—'descent with modification'" or that "all organisms are related by ties of genealogy or descent from common ancestry along the branching patterns of life's tree." I use evolution in its second, stronger sense of "ties of genealogy" via direct heredity or direct parentage.

eukaryotic cells. The early cells engulfed (swallowed without digestion) the bacteria and the novel combination allowed the host cells to harness sunlight for energy and led eventually to the development of modern plant forms.[2]

Evolution via combination is by no means rare in biology. And it is the main mechanism, I believe, in another large system that evolves: technology. Novel technologies combine or are assembled from other, previously existing technologies and often become building blocks for creating yet further novel technologies. A global positioning system (GPS) provides location information by combining satellites, atomic clocks, radio transmission, and mathematical algorithms, all of which existed as earlier technologies. Atomic clocks aboard the satellites provide extremely precise time, and by measuring the time delays among signals received from multiple satellites, navigators can determine their position with high accuracy. We can say that GPS has its origins in—has its "parentage" in—satellites, atomic clocks, and radio receivers, and these in turn have parentage in the technologies that created *them*. We can also say that GPS becomes a fundamental building block of technologies such as modern ship and aircraft navigation. Notice there is a mechanism of heredity here, but it isn't variation and selection. GPS does not derive from successive variations of previous navigation systems; rather, it derives from combination.

In my book *The Nature of Technology: What It Is and How It Evolves*[3] I described this mechanism and called it *combinatorial evolution*. I wrote about it largely as a process within technology, while mentioning that it occurs in other fields as well—mathematics, for example. On deeper investigation I have

[2] This idea was first proposed by Margulis (1970) and is now widely supported (see Keeling 2010).
[3] This chapter draws heavily from this book (Arthur 2009).

discovered there is a small but significant literature on evolution by combination: mostly in biology, in the history of chemical metabolism, and in complex systems (e.g., Simon 1962; Margulis 1970, 1981; Smith and Szathmáry 1995; Holland 1975; Kauffman 1993; Wagner 2014).[4] I find this literature fascinating, but I also find it scattered and more than a little restricted. For the most part, writings are specific to the particular domain they are describing, and they talk about new entities—novel structures within cells, new metabolic reactions, new pattern formations—being created via combination and therefore pushing evolution forward by one step or at most two. Missing is a description of a general, continuing process of evolution being driven by combination.

In this short overview, I want to provide such a description. In particular I want to introduce a certain interesting class of systems in which this form of evolution is present. I will use technology to illustrate how the mechanism works and will briefly indicate how it shows up in other fields.

A Class of Systems That Show Combinatorial Evolution

To set a context for combinatorial evolution, I want to talk about a class of systems that are at base collections of elements. Language is such a system; it contains a collection of elements—vocabulary words and punctuation symbols—that can be combined to form expressions—sentences or speech utterances for immediate purposes—without limit. Wilhelm von Humboldt ([1836]1988, 91) famously observed that language allows for the "infinite use of finite means." A quite different system, a coinage system,

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[4] In particular, Herbert Simon's paper of 1962 is a classic and it anticipates some of the ideas here. It shows that complex systems may often evolve in a hierarchical way by integrating or combining simpler constructions (intermediate forms) into higher-level ones. This promotes modularity, enables complex systems to evolve more rapidly by reducing the overall complexity of changes needed, and allows such systems to survive the transition providing the intermediate forms are stable.

similarly contains a collection of elements: coins of different denominations, which in combination can produce arbitrary cash amounts for payment or for tendering change. This system also allows for infinite use of finite means. Languages and coinage systems are examples of *combinatorial systems*: They have elements—building blocks if you like—that can be combined into *expressions* or *utterances* for immediate purposes, much as LEGO brick elements can be used to form toy-model constructions for immediate purposes. Combination follows a set of allowable rules or *grammar*, and we can think of the system as a *collection* or *toolbox* of reusable element types that allows new and purposed combinations to arise. If the system is one in nature, combination might happen haphazardly with not all results having a useful purpose or conforming to the grammar; if it is directed by human design, likely the combination will be properly formed and comply with the grammar.

Some combinatorial systems contain a fixed set of unchanging element types that can be used repeatedly for different immediate purposes, but do not evolve. I want now to go beyond this and look at combinatorial systems where *novel* element types can be generated and added to the toolbox from time to time. These, too, are formed from existing elements via combination. This may happen randomly, or it may happen because a particular combination is found to be useful repeatedly, or it may happen because a new element answers some ongoing need. Whatever the case, new elements may be generated from time to time and added to the toolbox, where they become available as new building blocks to form expressions and create further novel elements. I will call such systems *generative combinatorial* systems, or simply *generative systems*.

As an example of a generative system, think of an (imaginary) computer language.[5] It has both a basic vocabulary of instructions and a library of often-used functions that carry out recurring tasks like calculating logarithms or plotting data. These library functions are prewritten code, included so that programmers don't have to write them each time they use them, and together with the basic instructions they form the toolbox of the language. Programming applications—"expressions" if you like—are constructed from the toolbox, and from time to time new library functions are added to it, also constructed from the basic instructions and existing library functions. These become building blocks available for immediate coding and for generating yet further library functions. In this way the language evolves. Of course, the evolution in this case would not be over eons, it would be over months or years. If we looked at the elements in a programming library we could construct a family tree of their ancestry back to basic instructions, but it wouldn't be a neatly branching tree; it would be a crisscrossing network because most library functions have multiple ancestors.[6]

We now have a class of systems to work with, but how exactly does *evolution* happen within such systems?

What really evolves in a generative system is its collection of element types. The steps are not particularly complicated, so let me delineate them here:

1. Some recurring or ongoing need (or opportunity) arises.
2. A combination is found or emerges that fulfills this need.
3. The combination becomes an element in its own right, and it becomes *encapsulated*, existing as its own entity. It then

[5] I say *imaginary* because actual computer languages have several libraries, and computer language design is by no means simple (Stroustrup 1994).

[6] For empirical evidence of this see Valverde, Vidiella, and Duran-Nebreda (2026).

becomes a member of the library or toolbox, available for further combination.

Step 1 means that some need or purpose arises that poses an open problem or creates an opportunity that demands to be fulfilled. In Step 2, a solution to this problem or opportunity is found, either deliberately or by showing up randomly. In Step 3, this solution or successful combination, if used often enough, becomes a module. It becomes "selected." It gets its own label and becomes encapsulated in a device, or method, or element available for further combination. Encapsulation gives us fresh building blocks, and these may or may not replace old elements. In this way the toolbox changes in composition as its elements change and provides for new and fresh immediate expressions. It adapts, in other words, by providing novel elements that reflect new needs in its changing environment. The overall system evolves slowly over time as its elements change and provide for new combinations.

The details of specific systems that conform to this overall description differ in each field where we find combinatorial evolution. Some may offer a toolbox that can contain only a limited number of elements. Some may evolve novel elements via variation and selection in addition to the mechanism above—in fact, occasionally significant new combinations arise via variation and selection. Some may generate novel elements that render previous ones obsolete and we can say then they have both active and inactive elements.

Notice that the systems I am describing operate on two timescales: a fast one where they create expressions or utterances for immediate use; and a slow one where they occasionally create novel element-types by combining previous ones. It is not hard to show that generative systems exhibit in common other interesting—and rich—features. I list these separately in Box 1.

Combinatorial Evolution in Technology

I have been talking generally so far, and in what follows I want to discuss actual examples of systems that show combinatorial evolution.[7] Let me begin with technology. By *technology* (singular) I mean simply a means to a human purpose: the steam engine, the railway locomotive, a statistical method, a sorting algorithm. By *technology* (plural) I mean the overall collection of such means. The context will make clear which is meant.

How does combination arise with technologies? Actually, it is something that arises naturally. Technologies are assembled from parts or subassemblies, and so by nature they are combinations.[8] There is a reason for this. To do its work, a technology uses a phenomenon, usually several, and the phenomenon must be set up, properly prepared, and properly controlled. This requires many parts.

A hydroelectric power plant, for example, uses water falling under gravity to turn huge turbines, which move generators whose rotating magnets move electricity in neighboring stationary coils of wire—this latter effect is its base phenomenon. But the water must be stored, channeled into intake pipes (called penstocks), and directed into the turbines that move generators so that electricity flows, and the electricity must be stepped up in voltage and fed into transmission lines. So the system is an amalgam of reservoirs, penstocks, turbines, stators, transformers, and high-voltage transmission lines. Hydroelectric power generation is

[7] For details on the ideas presented here, see Arthur (2007, 2009). On technology see also Koppl *et al.* (2023).

[8] This notion that technologies are combinations of other technologies goes back at least to Thurston ([1878]1971) and has been proposed by several authors since, such as Usher ([1929]1988) and Gilfillan (1935).

Box 1. Generative combinatorial systems exhibit certain common properties.

No upper limit. Expressions and building blocks that can be created are in principle infinite in number.

Perpetual novelty. Such systems can continually create expressions and building blocks that have not been used before; they are open and never complete.

Two timescales. On a fast timescale, such systems create expressions—utterances—by routine combination of their toolbox elements for immediate purposes. On a slow timescale they generate new toolbox elements by occasionally combining ones that already exist.

Recursiveness. Elements are formed from subparts that are combinations of elements formed from subparts . . . . Each element forms a hierarchy of combinations down to individual basic elements.

Changing internal composition. Individual elements may alter their internal composition over time. An electric car contains a battery subsystem, which may change and improve over the life of the vehicle. Generative systems are fluid, highly configurable, and changeable over time.

Autopoiesis. Such systems create novel elements from existing elements in a self-sustaining way. Technically we can say that they are *autopoietic*[a]—they create themselves out of themselves.

Splintering. Such systems or toolbox collections easily splinter into special subsystems with their own methods or grammar.[b] Chemistry splits into inorganic chemistry, organic chemistry, biochemistry, nanochemistry, and so on. Each of these may become its own specialty.

An ecology. Elements within systems often form an ecology: They may support each other, compete with each other, negate each other. And often they interact with their outside environment.

Adaptation. Such systems adapt by generating novel building blocks that better conform to the needs of their current environment.

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[a] The term *autopoietic* was introduced in another context by Varela, Maturana, and Uribe (1974).
[b] For evidence of splintering (or splitting) in technology, see Lafond and Kim (2019).

an individual technology put together from a large toolbox of technologies.[9]

On a day-to-day basis technology or its toolbox libraries make possible individual constructions, expressions or utterances such as the Airbus A350, or the Hutong Yangtze River cable-stayed bridge. Technology is a language in everyday use. Its projects may be mundane or imaginative and elaborate.

So much for technologies and the toolbox they create. We still need to ask how novel toolbox elements in technology are generated. This is equivalent to Darwin's question of how species form. Some may indeed arise from variation and selection, and this is by no means infrequent (Basalla 1988). But if the novel technology—radar, GPS, the polymerase chain reaction—is radically new, it does not come from successive modifications of earlier technologies. It arrives independently in its own right. We are really asking here how *invention* happens. There is a literature on this from the 1930s,[10] but it isn't very satisfactory. Explanations of invention here inevitably invoke at the heart of the process some "act of insight" or "fortuitous configuration . . . in thought," but the fortuitous act is never quite spelled out.

My own explanation (Arthur 2007) is that invention is really a form of problem-solving, and as such is not particularly mysterious. It follows a distinct process. When some recurring or ongoing need arises, certain people—I call them *originators*—begin to look for a means to solve it.[11] They consider an overall

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[9] There are many such toolboxes, because technology tends to splinter. Individual technologies are based on particular families of phenomena—optical ones, chemical ones, electrical ones, electronic ones, digital ones—so we have optical technology, chemical technology, electrical technology, electronic technology, digital technology, and so on.

[10] See for example Usher ([1929]1988), Kaempffert (1930), and Gilfillan (1935).

[11] Invention may also proceed via harnessing new phenomena such as X-rays; here a similar process works (Arthur 2007).

principle—the idea of some phenomenon in use—in fact usually several principles, keeping in mind how each could be set up with the pieces or functionalities they have at their disposal. As they explore this, difficulties or hindrances arise and become subproblems that need to be resolved. These, too, need solutions, and so the process goes back and forth between problem and subproblem until a satisfactory solution has been reached. In 1972, when Gary Starkweather at Xerox Parc was looking for a means to print images from computers, he mulled over several candidate principles. One was to use a cathode ray tube to "burn" text onto paper surrounding it. A different, promising one was to use the computer to direct a laser beam to "paint" images on a copier drum; highly focused images could then be rolled off the copy machine. In the course of pursuing this principle several subproblems arose. One was that the lasers of the time were heavy and cumbersome and couldn't be moved rapidly enough to scan back and forth across the drum. Starkweather resolved this by keeping the laser fixed and having it direct its light at a revolving set of mirrors so that the beam could scan across the drum much as a lighthouse's beam scans across the horizon. After lengthy effort getting the individual components to work and resolving subproblems, the laser printer emerged (Starkweather 1980). It was the direct offspring of the computer, the laser, several optical technologies, and xerography.

Invention, then, at base is problem-solving using resources at hand. And of course it requires human agency (at least for now). It requires imagination, the ability to envisage some principle in use and the parts that might be needed for it, and it requires deep engineering skills, but it does not require genius or mysterious insight.

Once a novel technology or invention comes into being, it may become encapsulated and added to the toolbox. It then becomes a new building block, a new "species" if you like, available for further combination. We are now firmly in evolutionary territory. The collective of technology evolves or builds out via its "parentage"[12] in the previous technologies it builds from, and this way the overall collection of technologies bootstraps itself upward from the few to the many and from the simple to the complex. In its early days, the transistor plus a few other electronic components produced simple logic circuits; these served as building blocks for more complex circuits like adders, registers, and control units; and these in further rounds of combination produced arithmetic, graphic, and neural-network processors. Each stage contributed new building blocks for future technologies, and in this way over several decades the complex emerged from the simple.

The reader might check whether technology fulfills the properties listed in Box 1.

An obvious question at this stage arises: What use is it to know that technology evolves—and that it evolves via combination? My answer is this. In both our day-to-day thinking and in economic thinking we tend to see "technology" either as some vague macro thing that just exists, or as a set of independent objects—railroads, the steam engine, radar, mass production—that arise independently and sporadically. If, however we see technologies arising out of previous technologies and making possible future technologies, we see where technologies have come from and how they relate and interrelate over time. We also become conscious that our economy is not something given but something constantly forming and re-forming at all levels. Since the nineteenth century, economists have been aware that technology viewed as a "means of production" makes possible—

[12] "Parentage" need not include all components of a new technology. Practically speaking, I think of it as the base technologies that made the technology possible.

creates the skeleton of—the economy (Arthur 2021). We therefore begin to see where our economy has come from, and that it is never fully predictable and never at rest.

Change is ongoing in all subparts of technology, at all scales, and at all times; and this means that change is ongoing in all parts of the economy, at all scales, and at all times. The economy is ever creating itself anew.

Other Fields That Show Combinatorial Evolution

I want to show now that combinatorial evolution occurs in other fields besides technology, though it may not be the main driver in these. I would caution the reader that I am not a specialist in these areas, so my treatments here will be necessarily brief and simplified. My purpose is to indicate how combination applies across a number of fields without delving into full complexity. Specialists may find detailed analyses in domain-specific literature.

> **Chemistry.** In multistep chemical synthesis, a chemical product may typically combine with another product to create a novel product, which combines with another product to create a further product. For example, ethene reacts with oxygen to form ethylene oxide, which can further react with water to produce ethylene glycol. Ethylene glycol can in turn react with terephthalic acid to form polyester. In this way more complicated molecular structures arise from simpler ones and create new molecular structures for addition to the chemical "library" (Carey and Sundberg 2007). It was by such a process—a very long, drawn-out one—that in ancient times metabolic chemistry bootstrapped its way from the early formation of simple organic molecules, such as amino acids and nucleotides, to the development of more complex molecules capable of catalyzing reactions. These molecules facilitated

the breakdown of nutrients for energy and the synthesis of new cellular components. This gradual increase in complexity led to the formation of protocells, which acted to encapsulate and concentrate metabolic reactions, and in due course these simpler molecules combined and organized into the complex metabolic systems we see in modern organisms (Wagner 2014; Luisi 2016). A contemporary version of combinatorial evolution shows up in the new field of combinatorial synthesis, where diverse building blocks are deliberately, experimentally combined to produce a high multiplicity of outcomes. Specific combinations are then selected based on their alignment with a target outcome (Beck-Sickinger and Weber 2002).

Biological Evolution. In biology, combination is by no means rare; in fact, it turns out to be surprisingly frequent. It arises in several mechanisms, of which I will highlight four: symbiosis, horizontal gene transfer, the evolution of genetic regulatory networks, and recombination.[13] With symbiosis, biological entities may coexist for mutual benefit and occasionally merge or combine to form new structures (Margulis 1970, 1981), and this may be the cause of certain major transitions in evolution (Smith and Szathmáry 1995). Mitochondria and multicellular organisms likely originated this way. With horizontal gene transfer, organisms—especially bacteria and archaea—acquire genetic material directly from different species or organisms with different

[13] Combination also "underlies the evolution of proteins, in that many proteins are constructed of structural motifs, which evolution has recombined in different ways to construct new proteins, just as [happens] with technologies" (Erwin 2024). Modern evolutionary theory of course includes other processes such as genetic drift, developmental bias, niche construction, and multi-level selection.

> lineages; this foreign DNA integrates into the recipient's genome and, if stable, can be inherited by subsequent generations. Over time, horizontal gene transfer can lead to the emergence of new traits or new phenotypes in populations (Ochman, Lawrence, and Groisman 2000). With gene regulatory networks, we can say that these are built out of circuits that form a toolbox from which elements are combined for different development purposes (Erwin 2024). And with recombination, large pieces of genetic material are reshuffled and freshly combined, and the variation that results may or may not be selected for. In this case our mechanism and the basic one of variation and selection overlap. This occurrence of the two mechanisms I've been talking about working together is frequent, especially in natural systems. Combination may involve a long series of intermediate steps and these steps can be regarded as variations on which selection acts (e.g., Szostak, Bartel, and Luisi 2001). Not all these mechanisms fit the generative model I described above, but at least roughly, symbiosis and gene regulation do.

These examples of combination in chemistry and biology are well known. In three other fields—conceptual ones—this is less the case. Here, combinatorial evolution also takes place but it does so more in an abstract, nonphysical way: Existing concepts are combined—possibly implicitly—into new, encapsulated concepts, and in turn these become available for further combination. This sounds more than a touch ethereal, but as a mechanism of heredity in our human world it is nonetheless real.

Consider these fields as examples:

Mathematics. If we take the main objects that form the bones of mathematics—concepts, structures, axioms, methods, theorems and their proofs—a few of these are elemental, but the vast majority are constructions made up from simpler objects. Arithmetic is built from an accepted number system, axiom system, and from methods for addition, subtraction and division. Algebra is built from arithmetic, inherits its methods, and adds its own. Linear algebra is constructed from elementary algebra, and so on. In mathematics, construction—combination—is central. Mathematics once in place is used for mundane purposes or expression especially in commerce, engineering, and science. Novel elements arise if they prove useful and become encapsulated as concepts in their own right. Encapsulation is central in mathematics. Theorems are logical arguments encapsulated, methods are procedures for manipulation encapsulated, concepts are definitions in terms of simpler objects encapsulated. Encapsulation makes the discipline tractable,[14] much as abstract terms make specialized discourse tractable.

Evolution, the process of novel objects arising from combination of existing ones, is most clearly seen in the construction of proofs. Describing Andrew Wiles's proof of Fermat's Last Theorem, mathematician Kenneth Ribet remarks, "You turn a page and there's a brief appearance of some fundamental theorem by Deligne, and then you turn to another page and in some incidental way there's a theorem by Hellegouarch—all of these things are just

[14] Bertrand Russell (1919) commented dryly, "The process of deducing logic from first principles is an arduous one."

called into play and used for a moment before going on to the next idea" (quoted in Singh 1997). Mathematics architects its structures from existing structures which are themselves architected from previous structures. It builds itself in response to the external world and from elements it already possesses.

Language. In language, evolution mostly shows up in its more gradual form of descent with modification: Vocabulary changes slowly in meaning, and usage changes too (Crowley 1997). But evolution by combination also shows up: From time to time language reduces (combines, if you like) a complicated set of concepts into a single, usable word or neologism.[15] The fairly new word "Brexit" denotes Britain's decision to leave the European Union; and the suffix *-gate*, as in Watergate, stands for minor government malfeasance. Of course Brexit isn't a fixed combination of other words; it is at best a loose conceptual assemblage of simpler unstated actions and concepts. But it is an encapsulation that saves words and therefore earns its place in the language.

Legal System. At its core the legal system is a set of concepts and precedents that derive from combinations of earlier concepts and precedents. Such legal synthesis means analyzing existing legal principles, rules, and doctrines, and integrating them to create new legal concepts or to develop deeper understanding of existing ones. Of course the law changes by other processes, and in this one "parentage" would be hard to establish, so here evolution does become ethereal. Still, I like to keep this mechanism in mind because it forms a key generator for the evolution of the law, and it

[15] The process is called *lexicalization* (Brinton and Traugott 2005).

> gives a sense of historical linkage and provenance to legal ideas.

The reader might think of other fields where combinatorial evolution applies.[16] In the above systems, earlier elements can combine to add to a library of elements available for creating additional elements. We can see each system simultaneously as a language, a chemistry, or a technology.

Discussion

If we define our combinatorial evolution mechanism as the creation of new "toolbox" elements in a system via combination, we can see from the examples I've given that this mechanism often goes along with the Darwin's basic variation and selection one. Sometimes, as with technology, combination is followed by variation and selection—once a primitive aircraft exists, variations on it follow. Sometimes, as in biology and chemistry, a fresh combination follows from many steps of variation and selection. And sometimes, as with horizontal gene transfer, the two mechanisms overlap. The exact relation between the two mechanisms doesn't matter that much. What matters in combinatorial evolution is that new "toolbox" or "library" elements for further use are created via combination.

Before we close, let me make a few more general remarks about combination.

Combination is rarely if ever some new thing or event accomplished in single stroke. In the natural realm, it nearly always requires exploration of the space of possibilities along with a series of intermediate steps that lead to the novel combination. In the human realm, it may involve random exploration—tinkering—but more likely it calls for systematic, tested choices among

[16] For applications in archaeology see Lehner (2023) and Laursen (2023).

alternatives. In both cases combination is not what is put into a system; it is what comes out. Of course, standard variation and selection also involve "exploration" and many experimental intermediate steps before a final result—a new species, say—emerges. The overall outcome, whether evolution by combination or evolution by variation and selection, is nonetheless real.

I said earlier that combination can be ordinary and mundane, but sometimes—especially in nature—it can be remarkable. New combinations, Philip Anderson pointed out in 1972, may differ in quality and possibilities from their constituent components: Water has a "wetness" that neither oxygen nor hydrogen atoms possess—wetness we can say "emerges" from combination. On reflection, I believe what is remarkable about combinatorial systems is not simply the emergent qualities they generate, it is the *world* they generate. Combination—call it creative construction—in the human realm makes possible the world of music and of literature; in nature it makes possible the world of biochemistry and of life.

One of the puzzles of Darwin's original mechanism is how incremental modifications can bring into being elaborately constructed forms. How does the mammalian eye emerge from "numerous, successive, slight modifications" (Darwin 1859, 189) of more elementary structures? This question has been greatly argued over, and largely resolved in Darwin's favor—a chain of small modifications can indeed bring about novel forms. Still, as an evolutionary mechanism, it is easier to see how combination can quickly generate elaborate structures. It has a large adjacent possible, meaning it can make large leaps, which can directly allow the emergence of complicated new forms. If combination has this advantage, why don't we see more of it in nature? The reason is that novel combinations are unlikely to be viable precisely because they are *not* slight modifications; they may be haphazard in form and unlike any object that preceded them, and therefore not well adapted to survive, whereas an incremental Darwinian change will be more likely to survive simply because it is close to something that already survives. In the human domain, by contrast, we can create combinations that do survive, so that here evolution is relatively swift.

One last point. The combinatorial mechanism I have described is a shorthand version of how evolution works. In a given system, both expression and generation of novel element types require an apparatus to make routine combinations and to generate novel building blocks, and this apparatus normally lies outside the system we are looking at. A programming language requires not just a set of allowable instructions and a library that adds to these; it requires human (or artificial) minds that write code for these. The code requires computational machinery for its execution, and additions to the library require judgment from experienced practitioners. Expression and generation do not happen within the language itself. The same can be said for even the basic variation and selection mechanism. It is easy to talk about, but its realization in a biological world of organisms and their ecosystems requires a panoply of molecular machinery for copying and correcting DNA, for transcription, gene expression, and the construction of organisms themselves; such machinery is very much the subject of modern biological research. Variation and selection are simple, but the apparatus to realize these in the natural world is not.[17]

[17] This panoply of machinery must of course itself evolve, which implies that some evolutionary systems may evolve "symbiotically with"—in tandem with—other evolutionary systems.

Conclusion

In this chapter I have been describing a mechanism that lies behind evolution in a number of systems. Novel elements are created by combining earlier existing elements, and they become available as building blocks for creating yet further novel elements. This defines a mechanism of heredity, so we can say the system evolves. We can also say in general that from original simplicity complexity grows—it too evolves.

Combinatorial evolution occurs naturally in a class of systems that routinely combine elements for certain purposes. This class includes chemistry, mathematics, computation, technology, language, and several parts of biology. Such systems operate on a fast time-scale: They create expressions—utterances if you like—by routine combination of their member elements for immediate purposes. And they operate on a slow timescale: They generate new member elements by occasionally combining ones that already exist. It is this set of changes on the slower timescale that we call *evolution*. Systems in this class show several common features: recursion, openness to novelty, adaptation, and autopoiesis. Their evolution is *organic* in that they build new elements on top of previous ones; *indeterminate* in that we cannot predict what novel elements will arise next; and *history dependent* because the occurrence of novel elements is partly random and the sequence of their arrival determines what happens next in time.

Our combinatorial mechanism in no way negates Darwin's original, beautiful perception of successive selected modifications leading to novel structures—novel species. Combination and natural selection operate alongside each other in many systems. Combinatorial evolution is not rare, and where we see it we become aware of a connected, deep, ancestral history that we did not see before.

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